Flux Freezing

Übung zur Vorlesung:
Einführung in astrophysikalische Plasmen

Blatt 7, 12.11. - 26.11.2018

1. Schreibe das Magnetfeld in der Form $\mathbf{B} = \nabla\alpha \times \nabla\beta$ für skalare Funktionen $\alpha(\mathbf{r}, t)$ und $\beta(\mathbf{r}, t)$ (Clebsch Koordinaten). Zeige, dass $\nabla \cdot \mathbf{B} = 0$ gilt und dass α und β entlang Magnetfeldlinien konstant sind. Zeige weiter, dass die Zeitentwicklung des Magnetfelds ${}^{\partial\mathbf{B}}/_{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$ durch die Advektion von α und β mit der Flüssigkeit beschrieben werden kann:

$$\begin{aligned} \frac{d\alpha}{dt} &= \frac{\partial\alpha}{\partial t} + \mathbf{v}\cdot\nabla\alpha = 0 \\ \frac{d\beta}{dt} &= \frac{\partial\beta}{\partial t} + \mathbf{v}\cdot\nabla\beta = 0. \end{aligned} \tag{1}$$

Verwende die Koordinatentransformation von x, y nach $\alpha(x, y, 0)$, $\beta(x, y, 0)$ und ihre Jacobideterminante, um den Fluss Φ durch eine beliebige Fläche in der xy-Ebene zu berechnen:

$$\Phi = \int (\nabla\alpha \times \nabla\beta) \cdot \hat{\mathbf{z}} dS = \int d\alpha d\beta. \tag{2}$$

Überzeuge Dich mit (1) und (2) von der Gültigkeit des 'Flux Freezing'.

2. Ein Beispiel für die Grenzen von Flux Freezing findet sich im Zusammenspiel von Jupiter mit seinem inneren Mond Io. Io ist sehr leitfähig und von Magnetfeldlinien von Jupiter durchzogen. Studiere das vereinfachte Modell, welches auf S. 59/60 in R. M. Kulsrud, "Plasma Physics for Astrophysics" beschrieben wird. Gib für alle Rechnungen in der Herleitung (welche meistens einzelne Koordinaten der Vektoren verwendet) die vollständige Vektorbeziehung an, welche für den Herleitungsschritt verwendet wurde. Leite mit der vorgängig in der Vorlesung diskutierten Zerfallszeit $T = 4\pi L^2/(\eta c)$ und $L \sim \sqrt{dD'}$ die einfache Beziehung $L = V_x T$ her. Was bedeutet dieser Zusammenhang anschaulich in Abbildung 3.7? Schätze aus den astronomischen Daten B_x/B_0 ab.

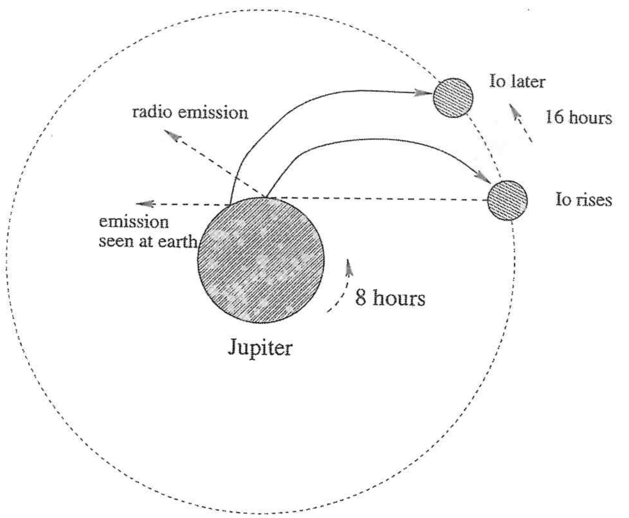

Figure 3.6. The relative orientation of Io, Jupiter, and the earth when radio emission is detected

Io

revolves around Jupiter in roughly 43 hours. Jupiter itself rotates faster, and in the same direction, with an equatorial period of 10 hours.

If the flux lines were completely frozen, both in Io and in Jupiter's ionosphere, and this ionosphere is rigidly attached to Jupiter itself, then the lines would be wrapped around Jupiter once every 13 hours and, thus, the field strength would quickly be amplified to an enormous value, roughly 12 G times the number of rotations. It is clear that flux freezing cannot hold, and that the lines must slip, either through Io or through Jupiter's ionosphere. In any event, the slippage gives rise to a large voltage. Thus, it is not surprising that the emission of synchrotron radiation, characteristic of electrons with energies of many kilovolts, is detected from the surface of Jupiter. This radiation tracks the relative motion of Io across the face of Jupiter (Duncan 1965). Being synchrotron radiation from nearly perpendicular-pitch-angle electrons, it is emitted primarily perpendicular to the magnetic field lines connecting Io and Jupiter. As a result, the emission is observed at the earth shortly after Io emerges from eclipse by Jupiter.

The slight delay can be understood from the geometry of the lines of force dragged by Io. Emission should be seen when the plane perpendicular to the line of force at its foot intersects the earth. The geometry of these dragged field lines when Io emerges from behind Jupiter is sketched in figure 3.6.

To understand what goes on during line slippage let us consider the grossly simplified model shown in figure 3.7. In this model assume that there are two horizontal infinitely conducting walls at $y = \pm(d + D')$, and that initially there is a vertical uniform field $B_y = B_0$. Let there be a horizontal rigid slab of thickness $2d$ moving with constant velocity $V_x = V_0$ in the x direction and centered about $y = 0$ (between $y = \pm d$). Let the open spaces between

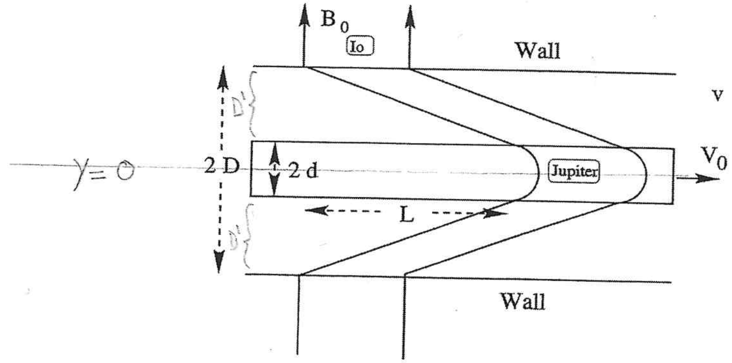

Figure 3.7. A conducting slab passing between two conducting plates as a model for the Io–Jupiter interaction

$B_0 \frac{L}{D'}$

the slab and the walls be filled with a very low pressure infinitely conducting plasma. Let the walls, the slab, and the field extend infinitely in the x and z direction. Let the slab have finite conductivity. What will happen?

If η is very small, the slab will first draw the lines to the right, satisfying $E_z = -V_x B_y/c$ inside itself. By symmetry B_y does not change, The dragged field lines will have a B_x component negative above and positive below $y = 0$. Consequently, at time t there will be a current density in the slab $4\pi j_z = -\partial B_x/\partial y \approx 2B_x/2d = -(L/d\cancel{D'})B_0$, where $L = V_0 t$ is the displacement of the slab. (Thus, $B_x \approx B_0 D'/L$.) As L increases, j_z increases until ηj_z becomes comparable to $V_0 B/c$ and the electric field approaches zero. That is to say, a steady state is reached.

To derive the steady state let us take as given that the current in the rarefied plasmas, in $d < y < d + D'$ and $-(d + D') < y < -d$, is zero, so $B_x =$ constant there. Then, above the slab

$$\frac{B_x}{B_0} = -\frac{L}{D'} \tag{42}$$

and below $B_x = LB_0/D'$. In this slab the z component of Ohm's law gives

$$\left(\frac{V \times B}{c}\right)_z = \frac{V_0 B_0}{c} = \eta j_z = -\frac{\eta}{4\pi}\frac{\partial B_x}{\partial y} \tag{43}$$

This is a constant because $V_x B_0/c$ is. Integrating this equation and matching to $B_x = \mp(L/D')B_0$ at $y = \pm d$, we find for $0 < y < d$

$$B_x = -\frac{V_x B_0}{\eta c/4\pi} y$$

and when $y = d$

$$B_x = \frac{V_x B_0}{\eta c/4\pi} d = -\frac{L}{D'} \tag{44}$$