Self-consistent gaps and equation of states for neutron star applications

Ding, Rios, *et al.*, *Phys Rev C* **94** 025802 (2016)
Sellahewa & Rios, *in preparation*

Arnau Rios Huguet
Lecturer & STFC Advanced Fellow
Department of Physics
University of Surrey

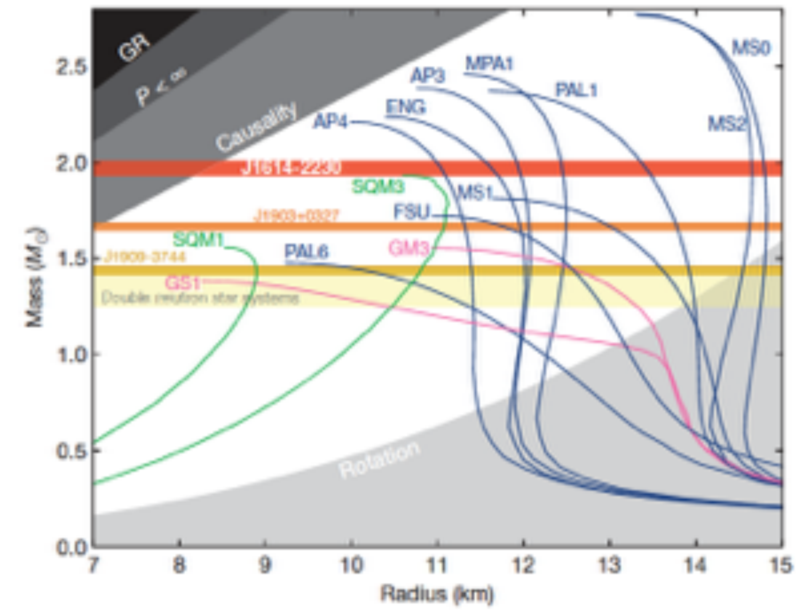
Basel Workshop, 29 September 2016

Neutron star modeling

Input #1
EoS

→

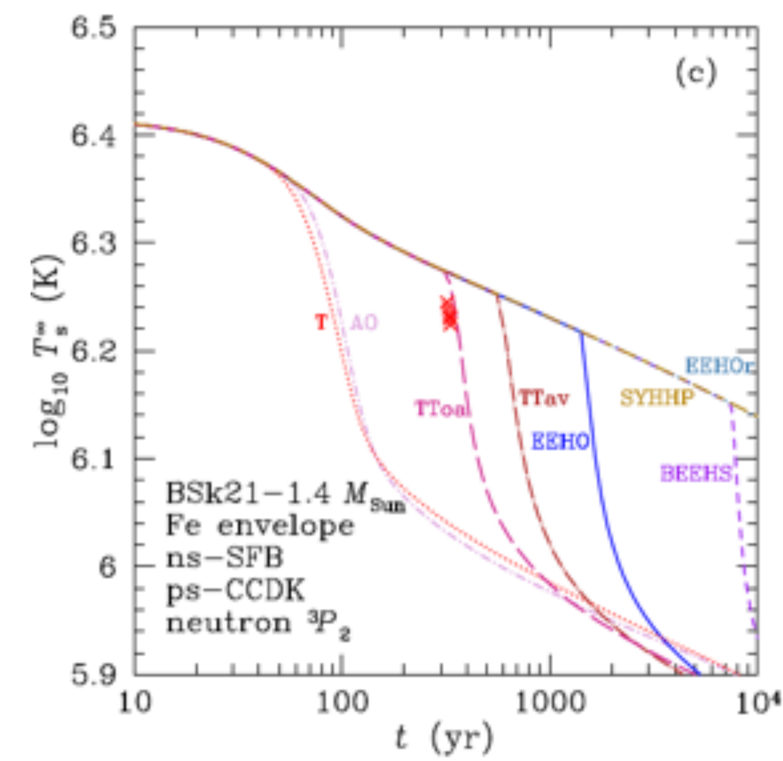
Observable #1
Mass-Radius relation



Input #2
Pairing gap

→

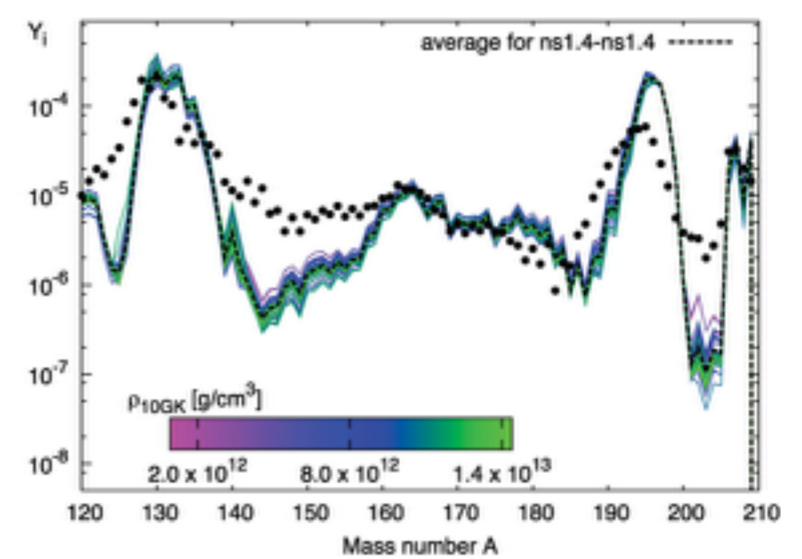
Observable #2
Cooling curve



Input #3
ν transport

→

Observable #3
Nucleosynthesis



Neutron star modeling

Input
Consistent many-body theory

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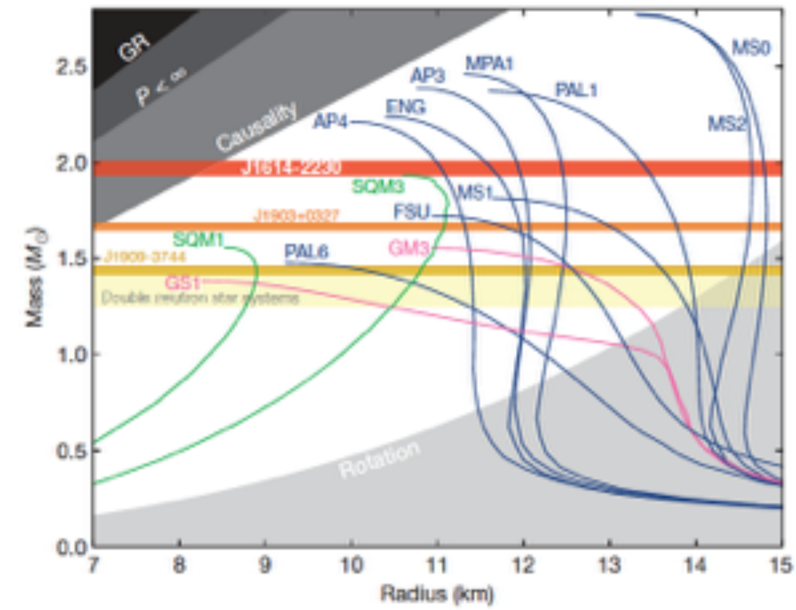
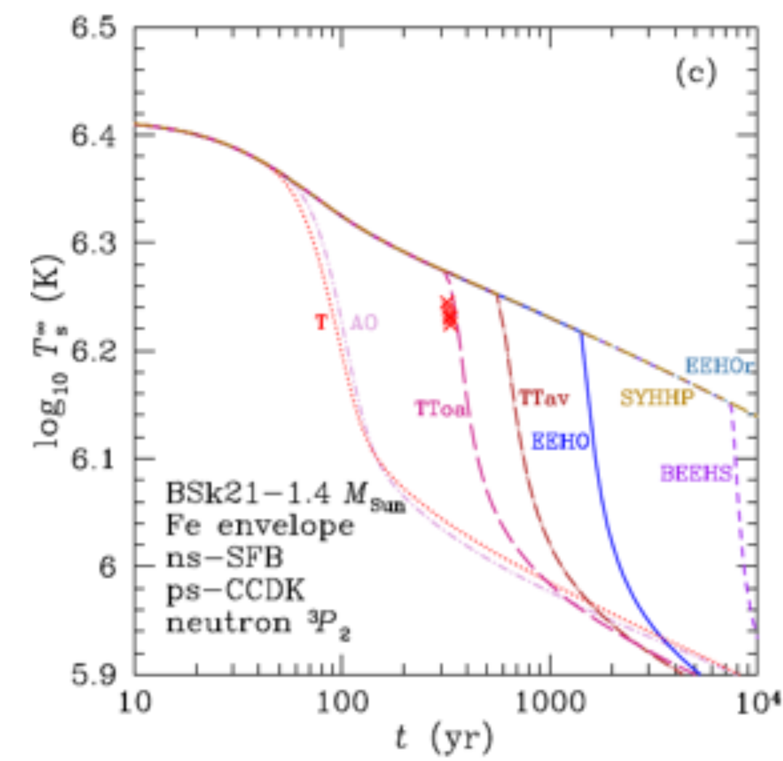


Observable #2
Cooling curve



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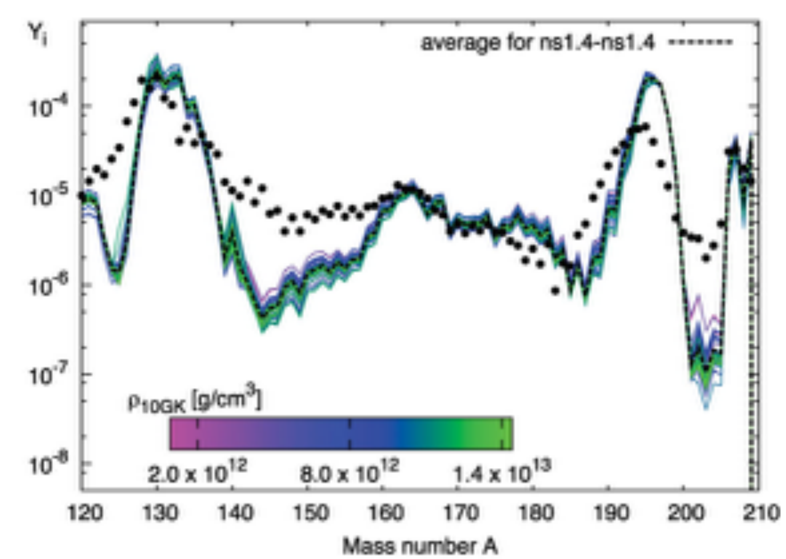

Neutron star modeling

Input
Consistent many-body theory

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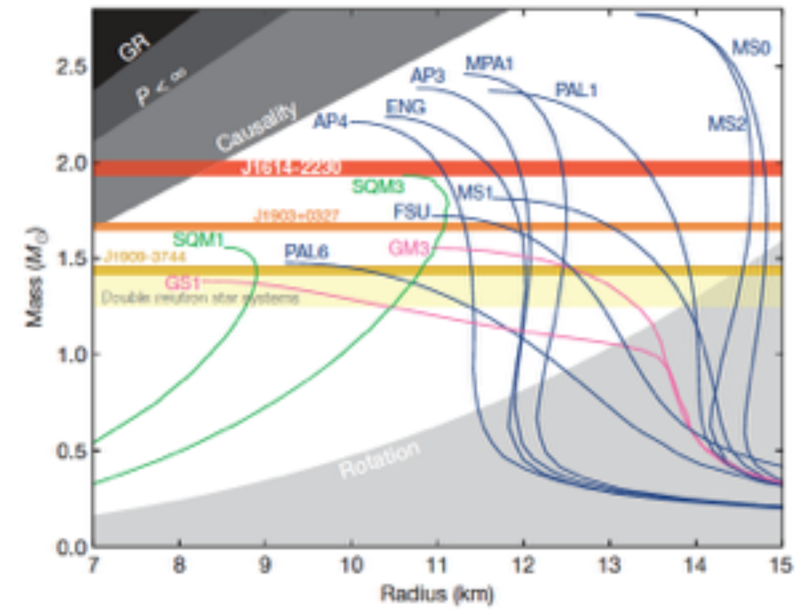
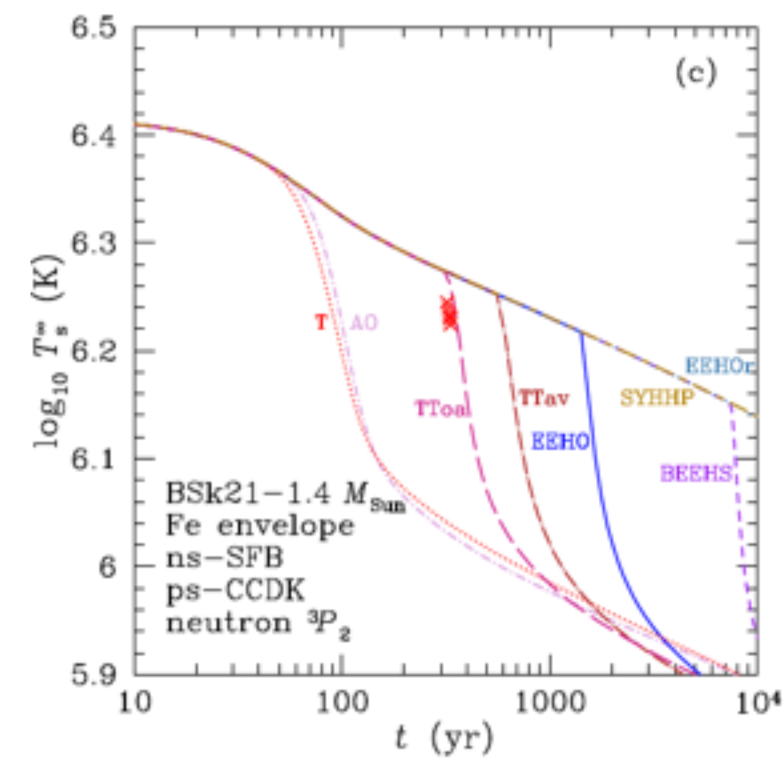


Observable #2
Cooling curve



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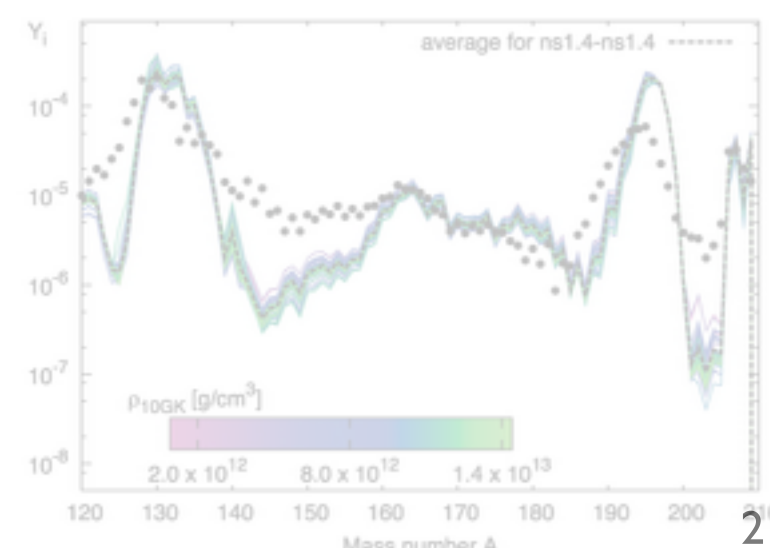


Summary

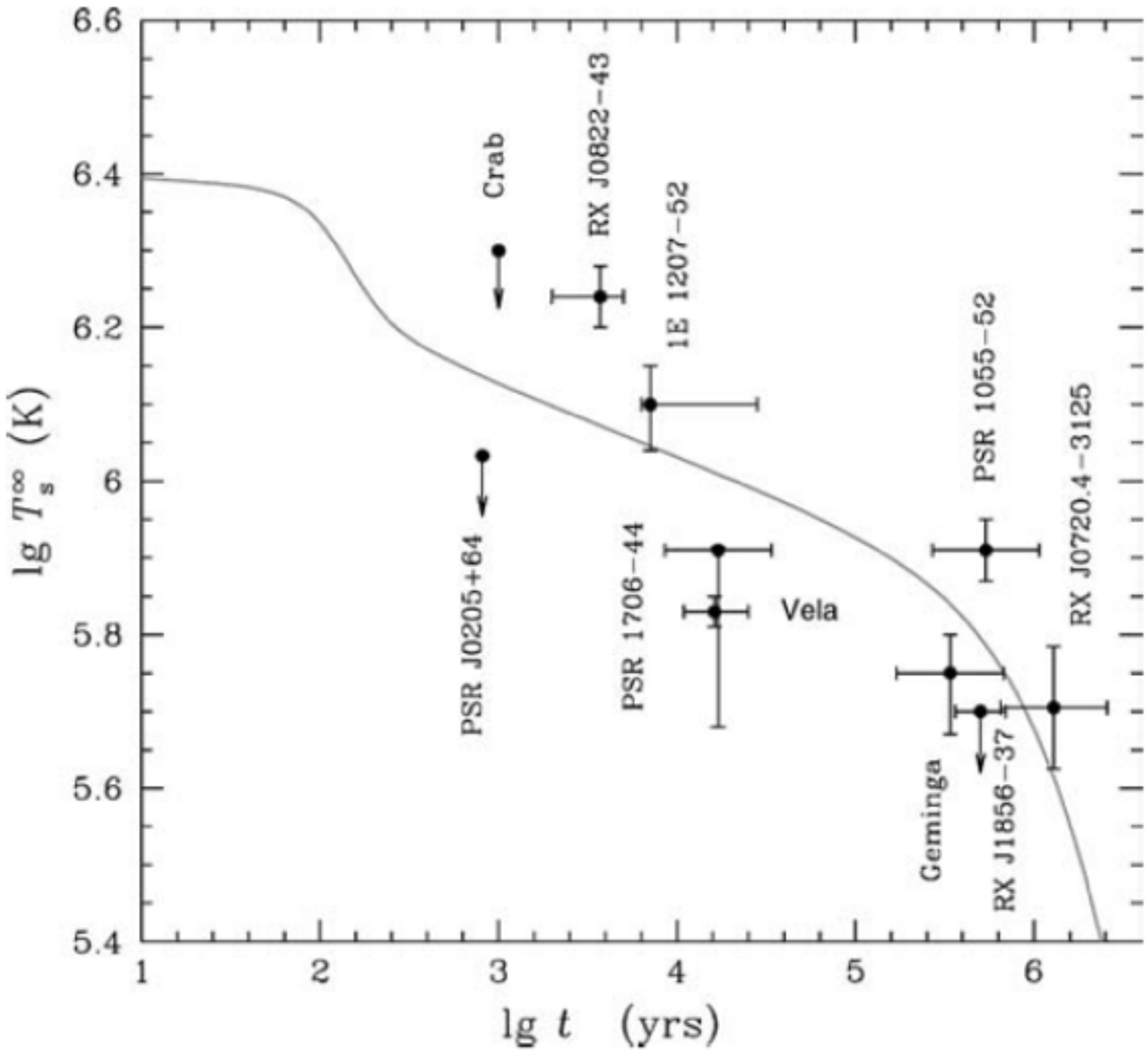
1. NS model with effective forces

2. NS model with realistic forces

3. Beyond-BCS pairing with systematic errors

Cooling of neutron stars

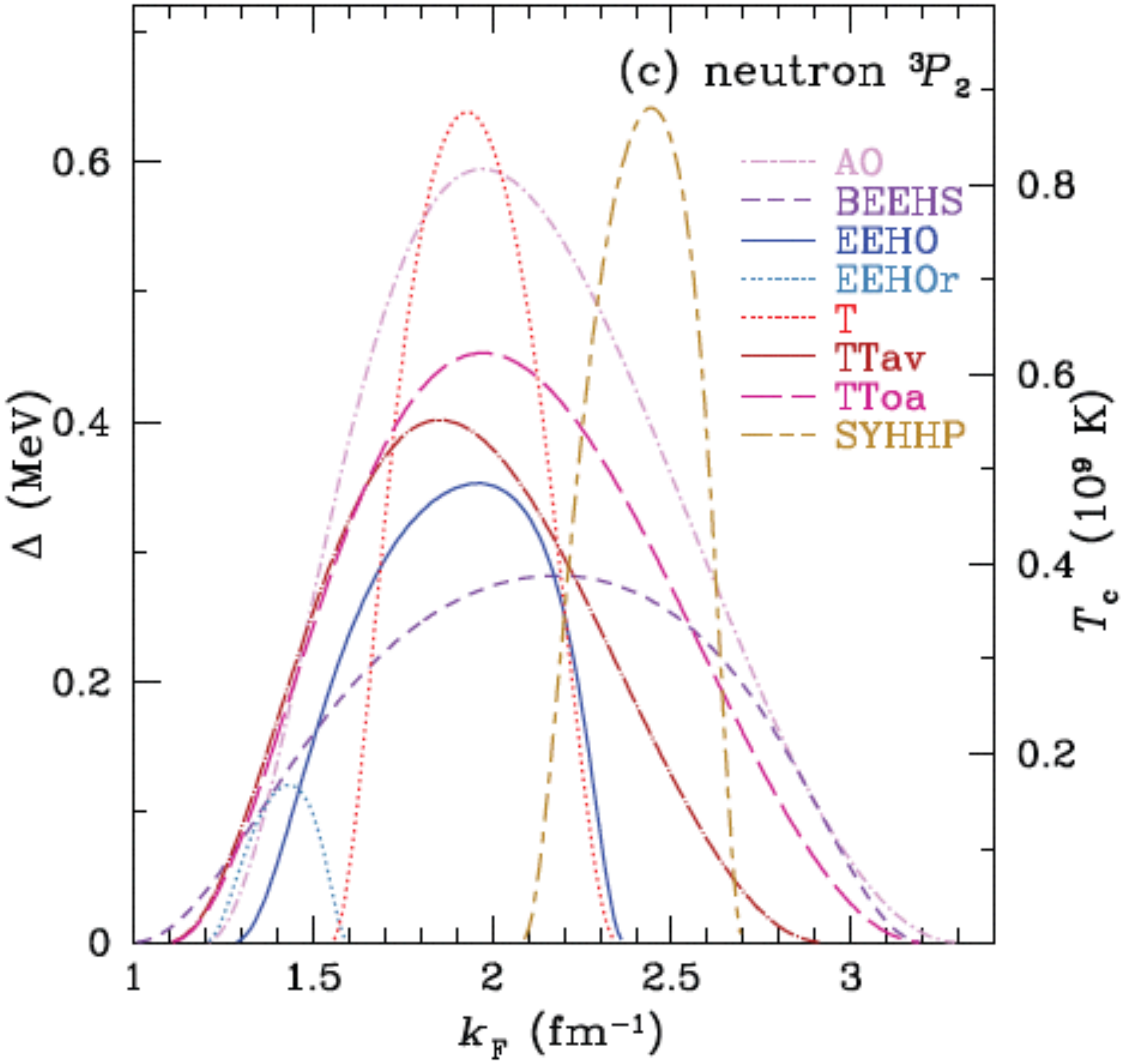
NS cooling curves ($M=1.3M_\odot$)



- Observational data available for a handful of NS
- Sensitive to **interior** physics!

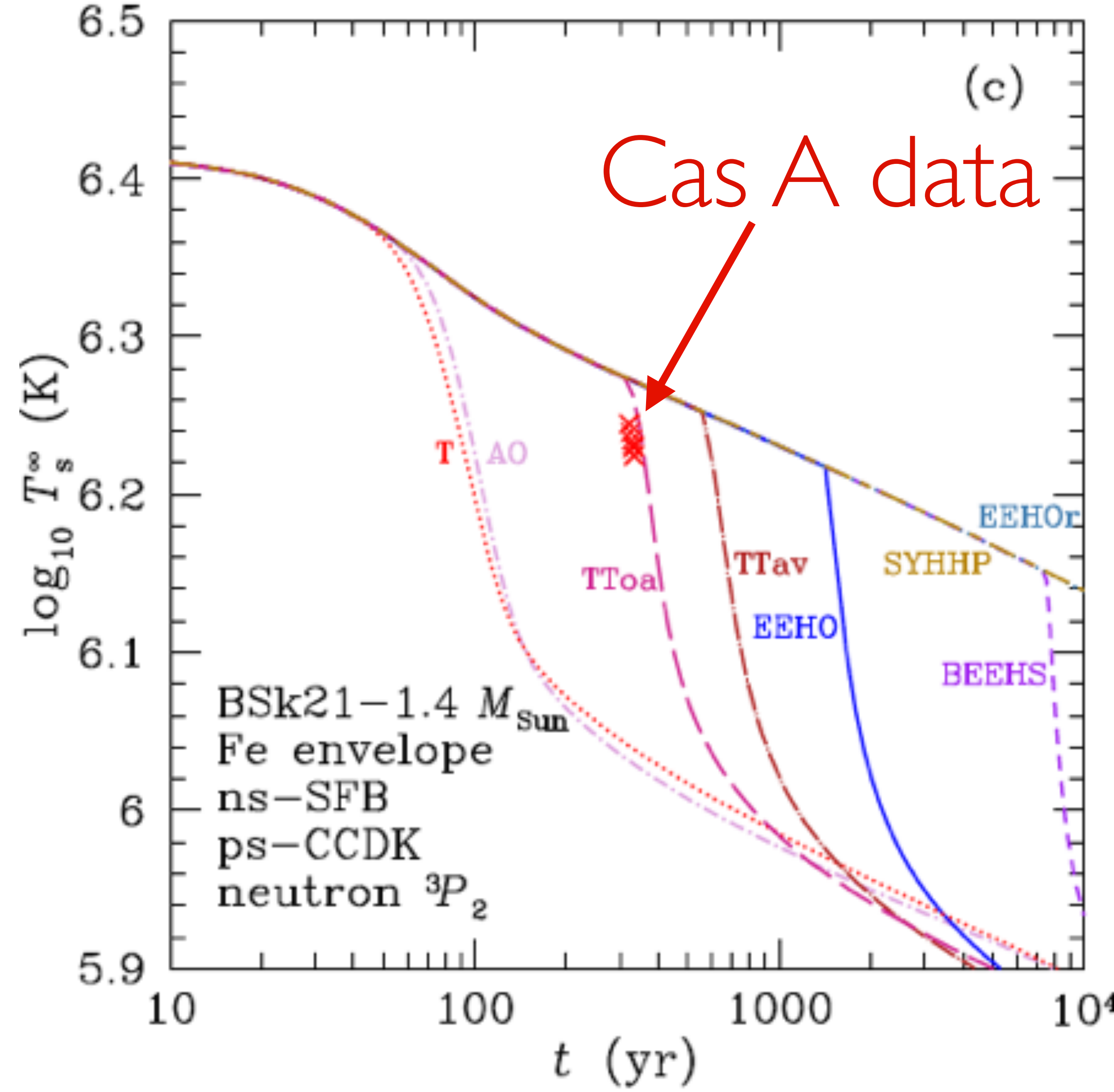
Yakovlev & Pethick, ARAA **42** 169 (2004)

Cooling of Cassiopea A



Ho, *et al.*, PRC **91** 015806 (2015)

Page, *et al.*, *PRL* **106** 081101 (2011)



Ingredients

(a) Mass of pulsar

(b) EoS (determines radius)

(c) Internal composition

(d) **Pairing gaps** ($^1S_0$ & $^3PF_2$ channels)

(e) Atmosphere composition

Name	Process	Emissivity (erg cm^{-3} s^{-1})
Modified Urca (neutron branch)	$n+n \to n+p+e^- +\bar{\nu}_e$ $n+p+e^- \to n+n+\nu_e$	$\sim 2{\times}10^{21}\,R\,T_9^8$
Modified Urca (proton branch)	$p+n \to p+p+e^- +\bar{\nu}_e$ $p+p+e^- \to p+n+\nu_e$	$\sim 10^{21}\,R\,T_9^8$
Bremsstrahlungs	$n+n \to n+n+\nu+\bar{\nu}$ $n+p \to n+p+\nu+\bar{\nu}$ $p+p \to p+p+\nu+\bar{\nu}$	$\sim 10^{19}\,R\,T_9^8$
Cooper pair	$n+n \to [nn]+\nu+\bar{\nu}$ $p+p \to [pp]+\nu+\bar{\nu}$	$\sim 5{\times}10^{21}\,R\,T_9^7$ $\sim 5{\times}10^{19}\,R\,T_9^7$
Direct Urca (nucleons)	$n \to p+e^- +\bar{\nu}_e$ $p+e^- \to n+\nu_e$	$\sim 10^{27}\,R\,T_9^6$

A step forward towards consistency

The Gogny force

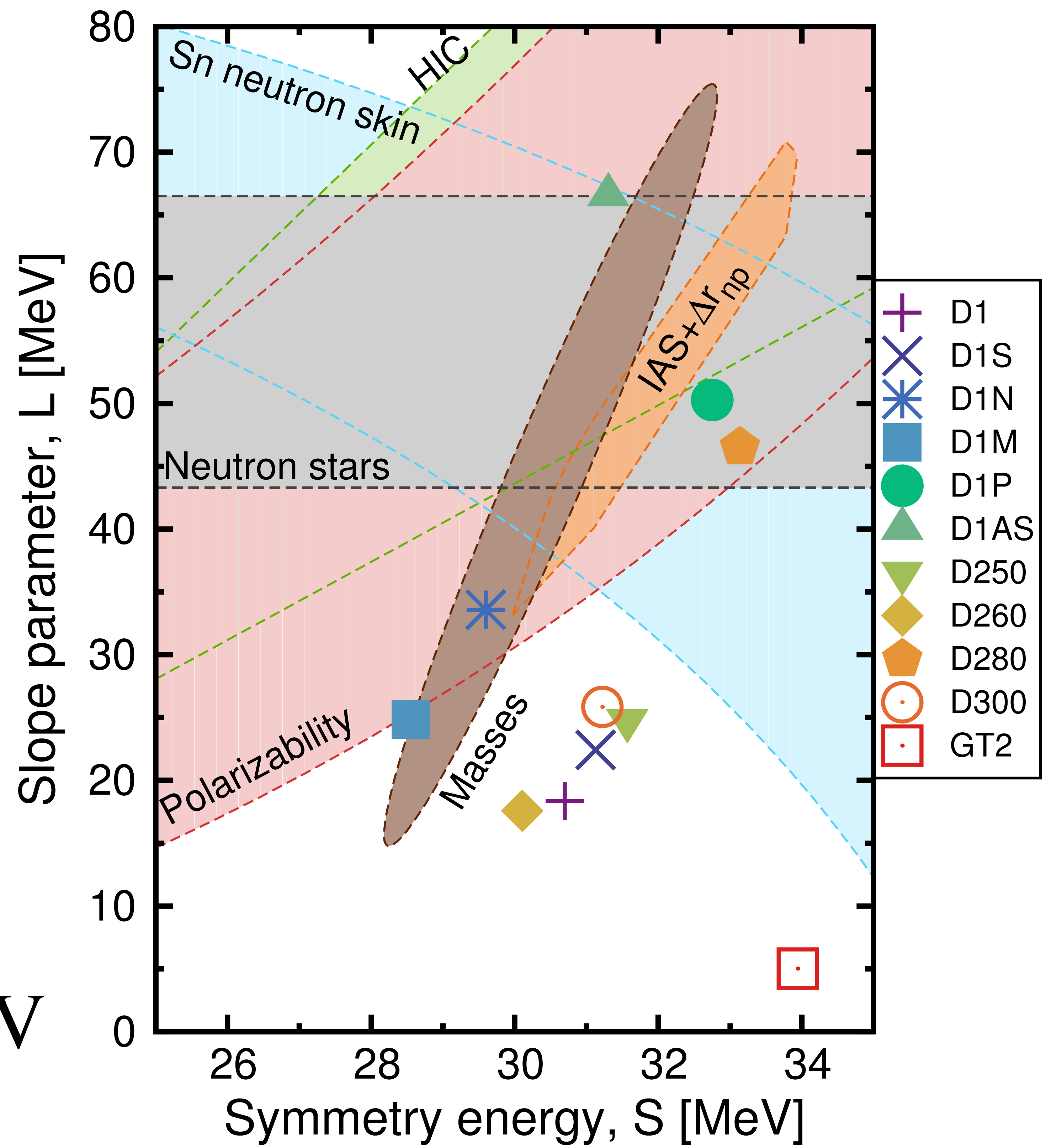
$$V(\vec{r}) = \sum_{i=1,2} \left(W_i + B_i\, P_\sigma - H_i\, P_\tau - M_i\, P_\sigma\, P_\tau\right) e^{-\frac{r^2}{\mu_i}}$$
$$+ \sum_{i=1,2} t_0^i\, \left(1 + x_0^i\, P_\sigma\right) \rho^{\alpha_i}\, \delta(\vec{r})$$
$$+ iW_0(\sigma_1 + \sigma_2)[\vec{k}' \times \delta(\vec{r})\vec{k}]$$

- 11 parametrizations
- Popular in **fission** studies

- Any good for isospin?
- Only 7 give stable NSs!

- **Pairing**?

S~31 ± 1 MeV
L~45 ± 10 MeV



compstar-esf.org

R. Sellahewa & A. Rios, *PRC* **90**, 054327 (2014)

A step forward towards consistency
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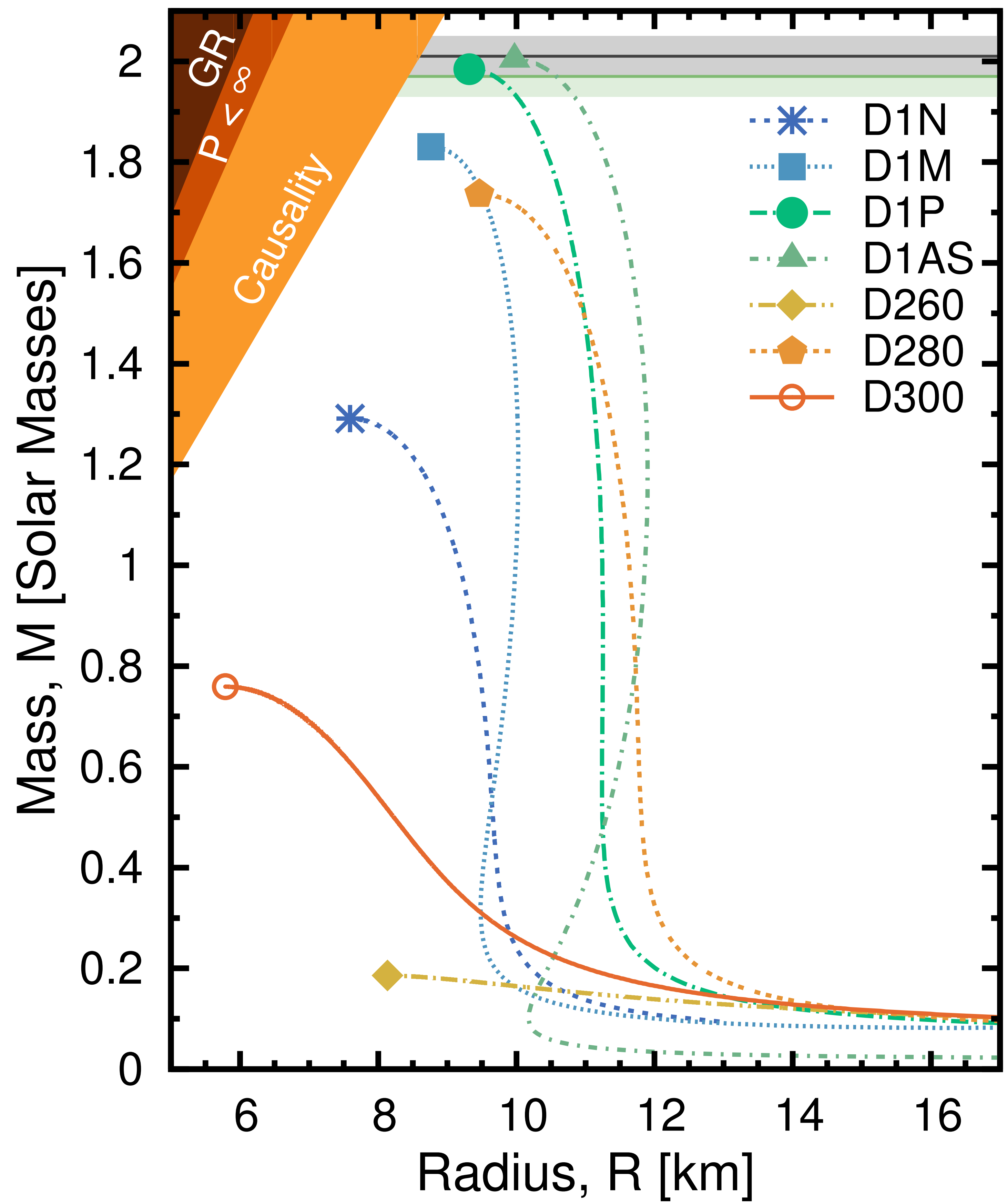
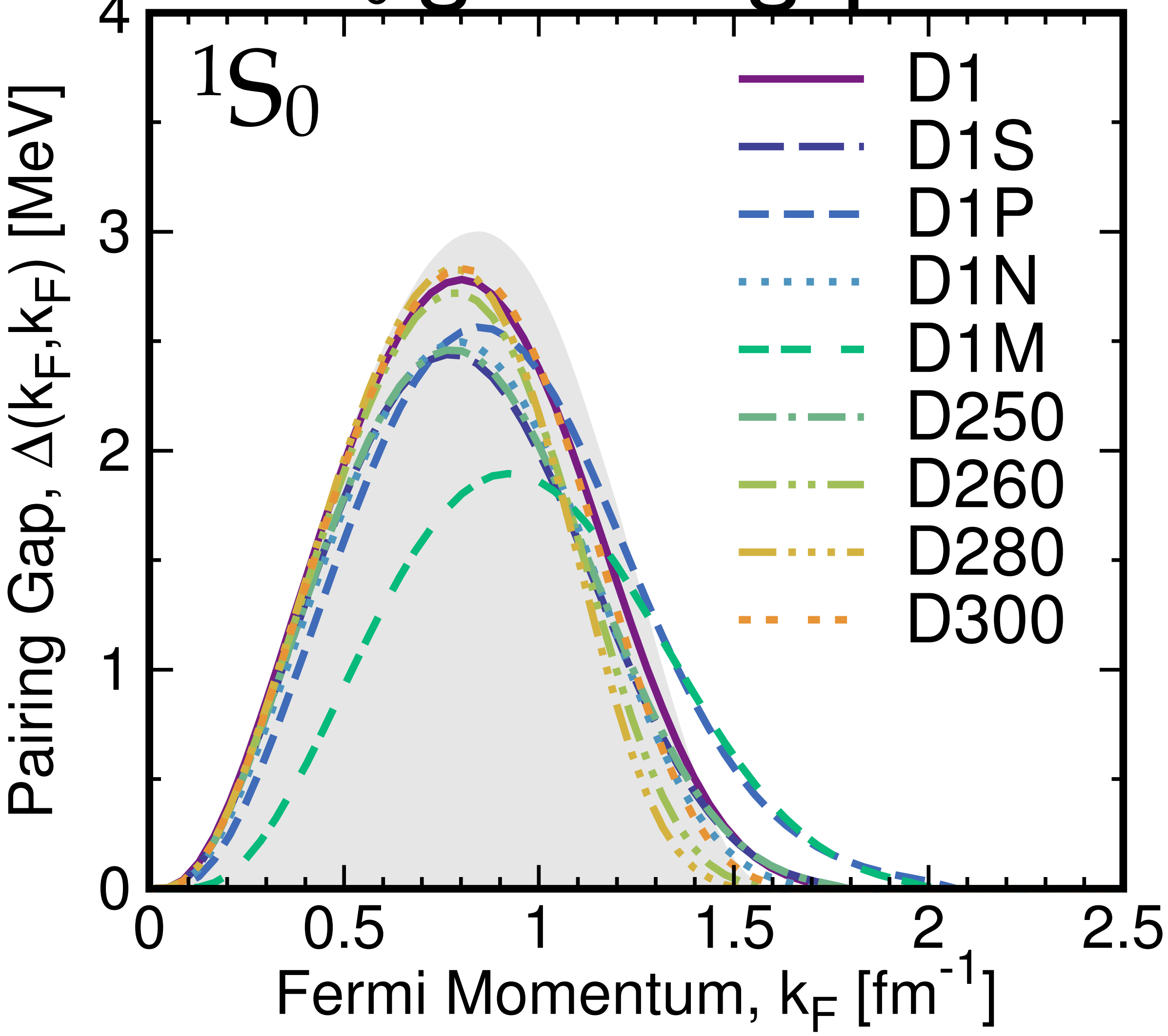


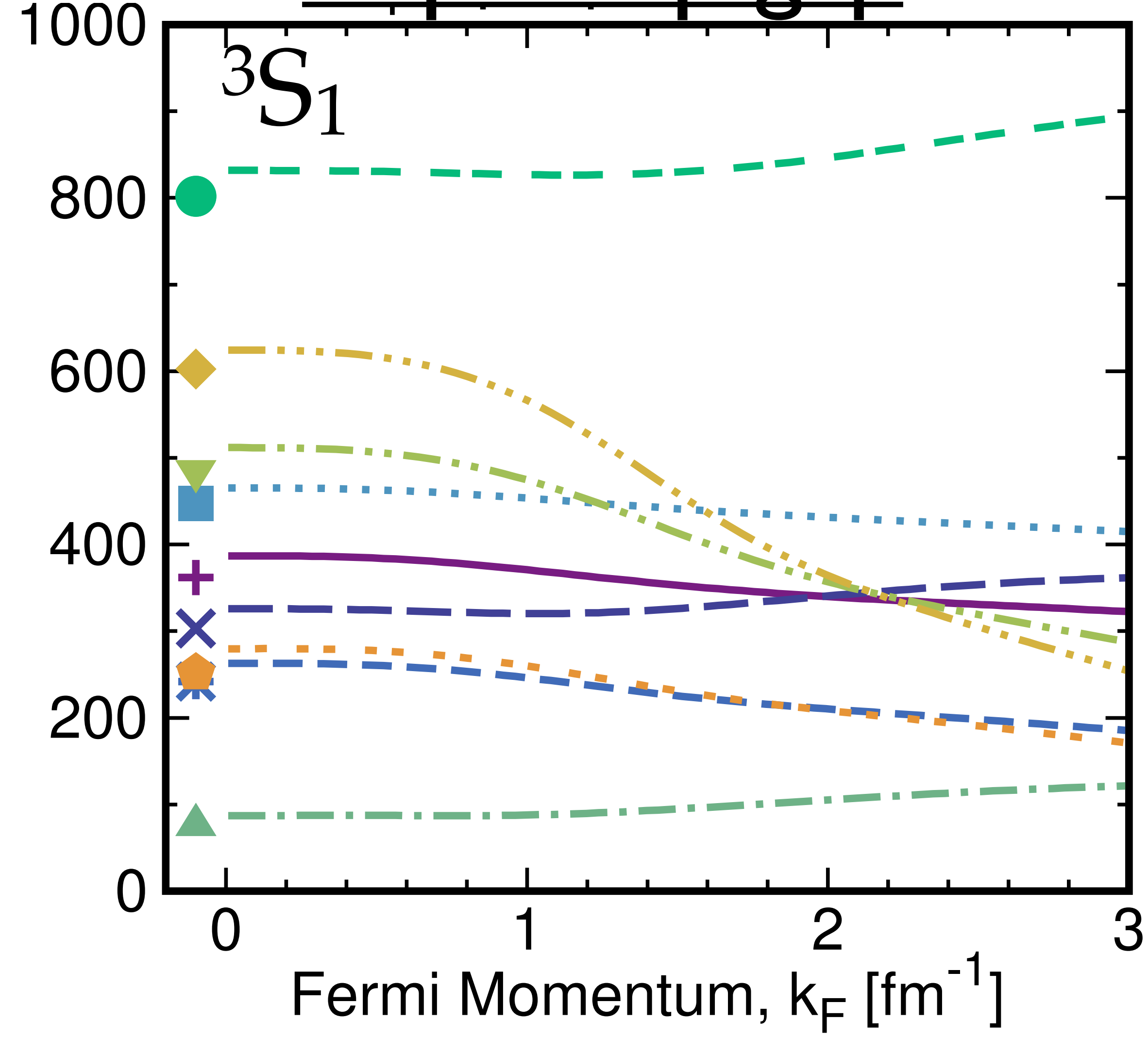
R. Sellahewa & A. Rios, *PRC* **90**, 054327 (2014)

BCS-pairing with Gogny

Singlet nn gap



Triplet np gap

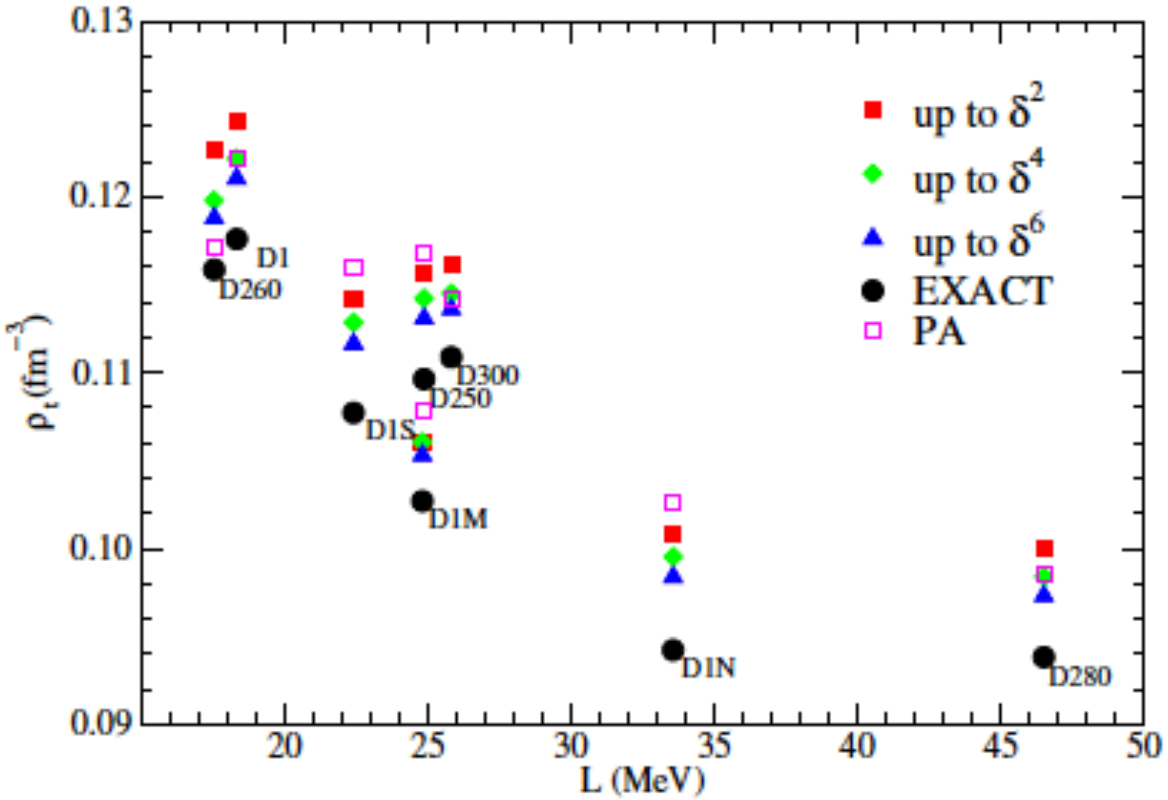


BCS equation

$$\Delta_k^L = \int_{k'} \frac{\langle k|V_{nn}^L|k'\rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}^L|^2}} \Delta_{k'}^L \quad \mathbf{+} \quad \chi_k = \varepsilon_k - \mu$$

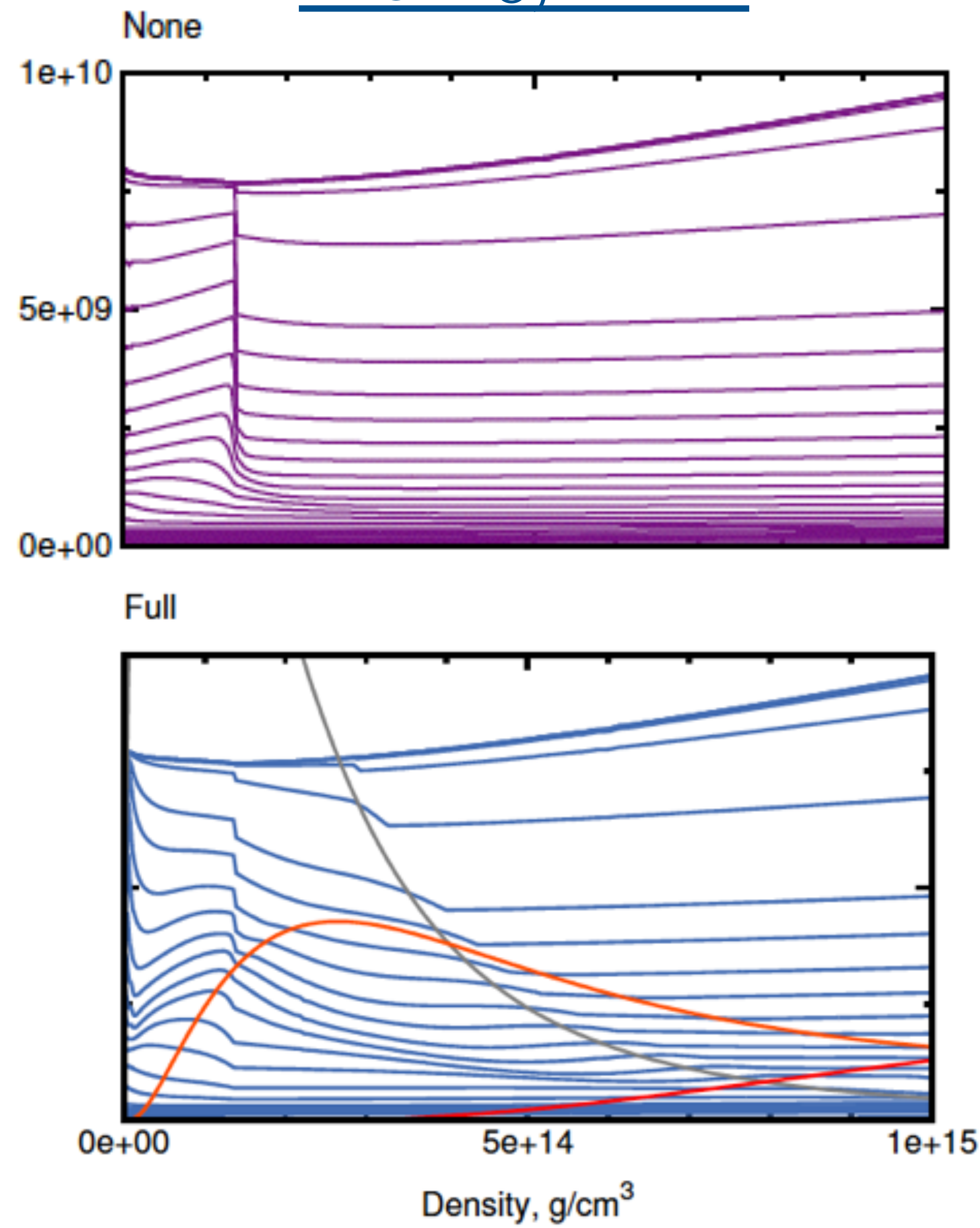
R. Sellahewa & A. Rios, *in preparation* + R. Sellahewa, *PhD thesis*

Cooling



Gonzalez-Boquera, Centelles, Vinas
& A. Rios, *in preparation*

1.6M⊙, D1P



Ingredients (D1P)

(a) Mass of pulsar, 1.6M⊙ ✔

(b) EoS (except crust) ✔

(c) Internal composition ✔

(d) Pairing gaps (n $^1S_0$ & $^3P_2$) ✔

(e) Atmosphere composition

R. Sellahewa, *PhD thesis*

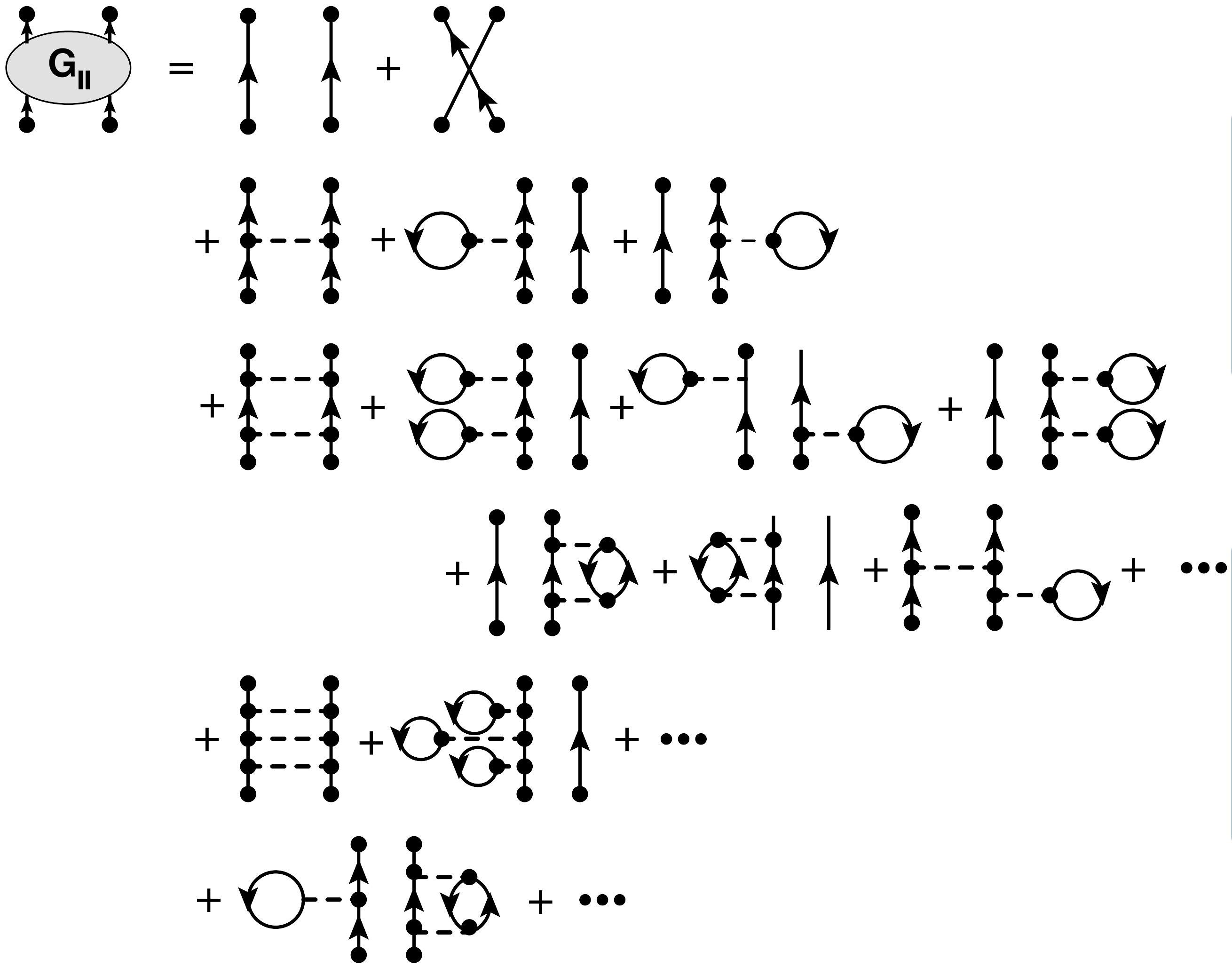
Summary

1. NS model with effective forces

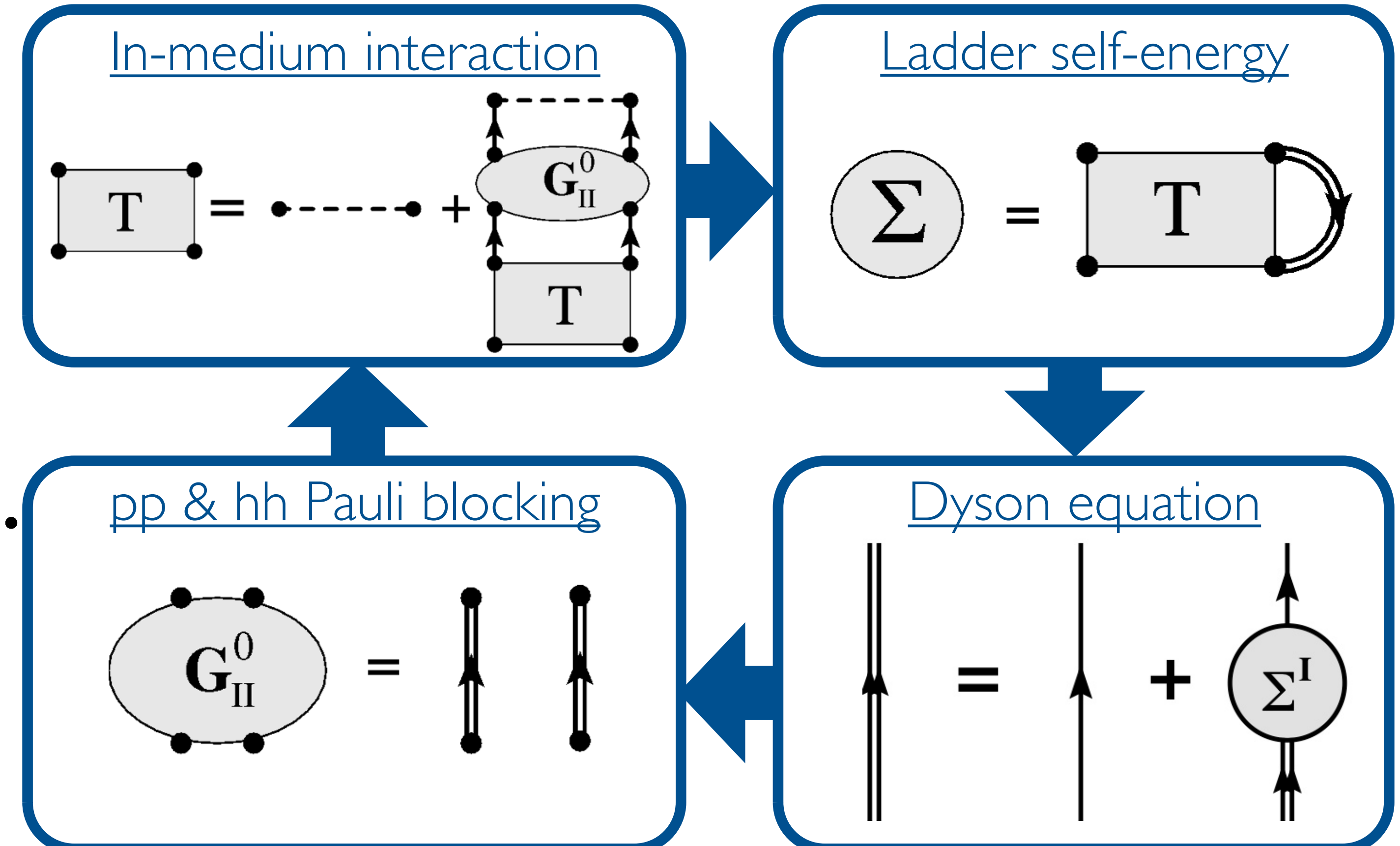
2. <u>NS model with realistic forces</u>

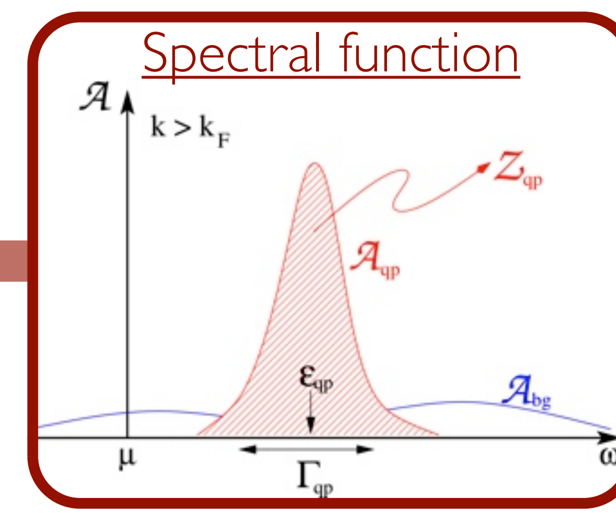
3. Beyond-BCS pairing with systematic errors

Self-consistent Green's functions



Ladder approximation within SCGF



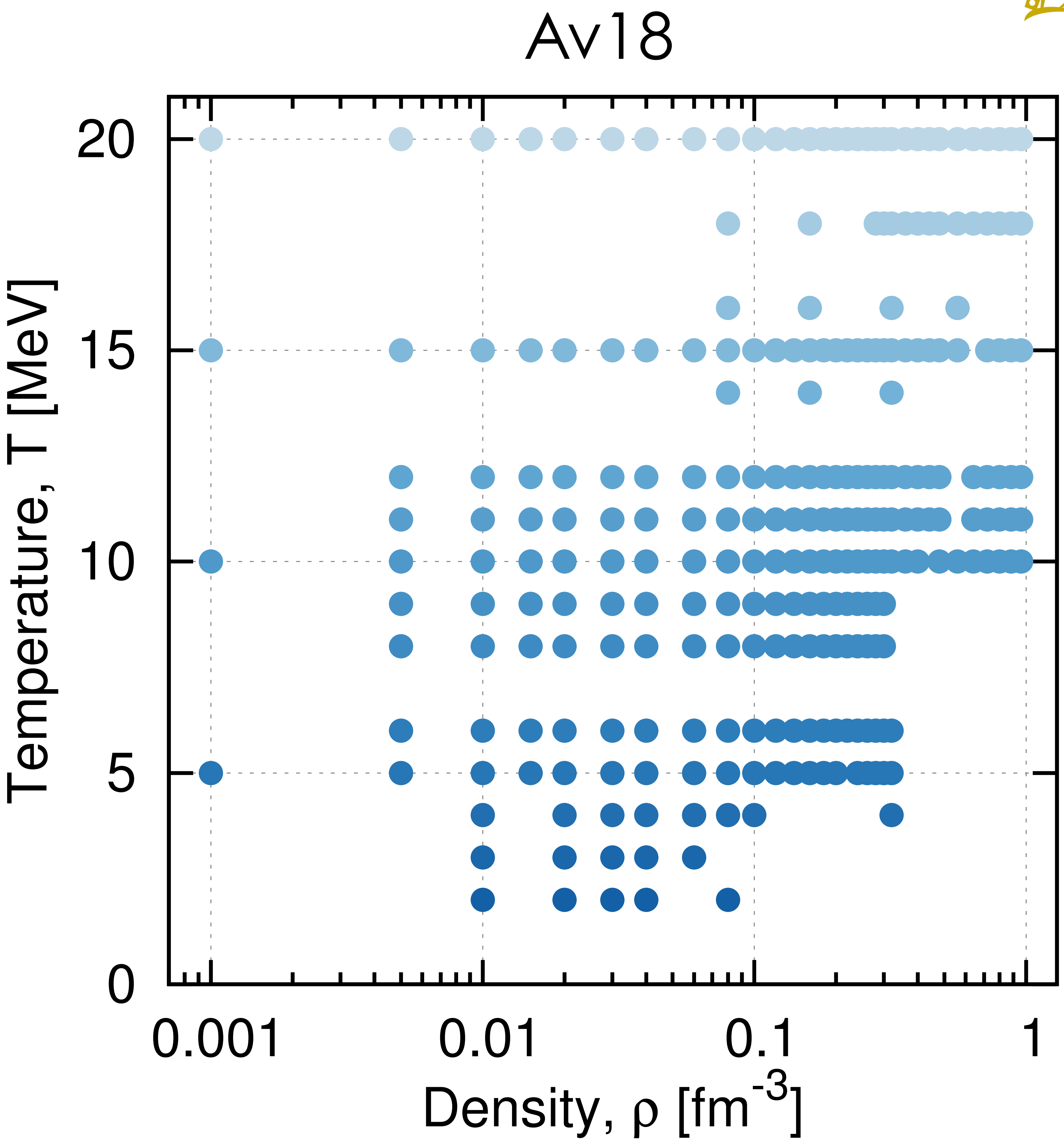


One-body properties
Momentum distribution
Thermodynamics & EoS
Transport

Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
Alm *et al.*, PRC **53** 2181 (1996)
Dewulf *et al.*, PRL **90** 152501 (2003)
Frick & Muther, PRC **68** 034310 (2003)
Rios, PhD Thesis, U. Barcelona (2007)
Soma & Bozek, PRC **78** 054003 (2008)

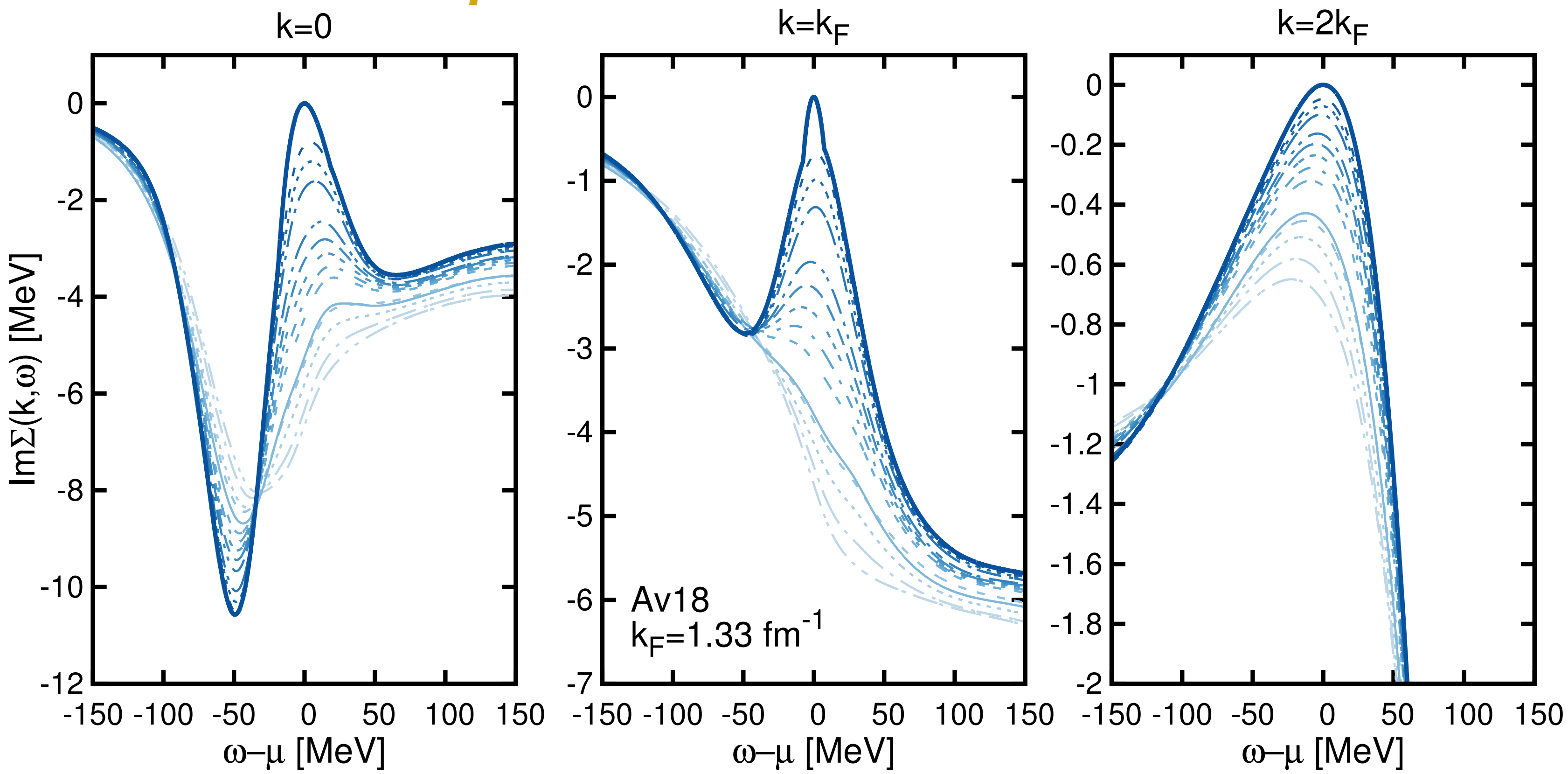
Self-consistency, pp+hh & full off-shell effects
Finite temperature

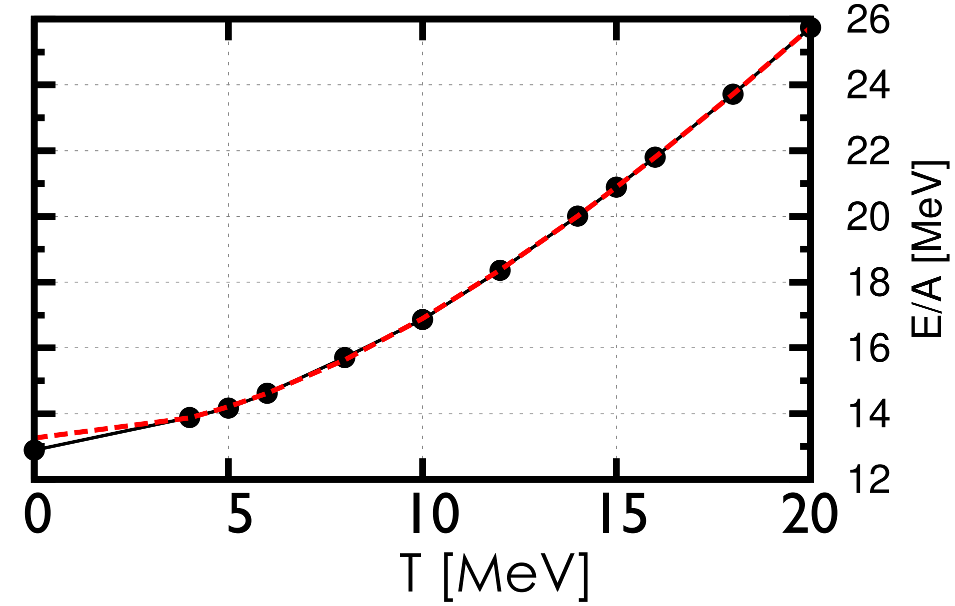
Available data

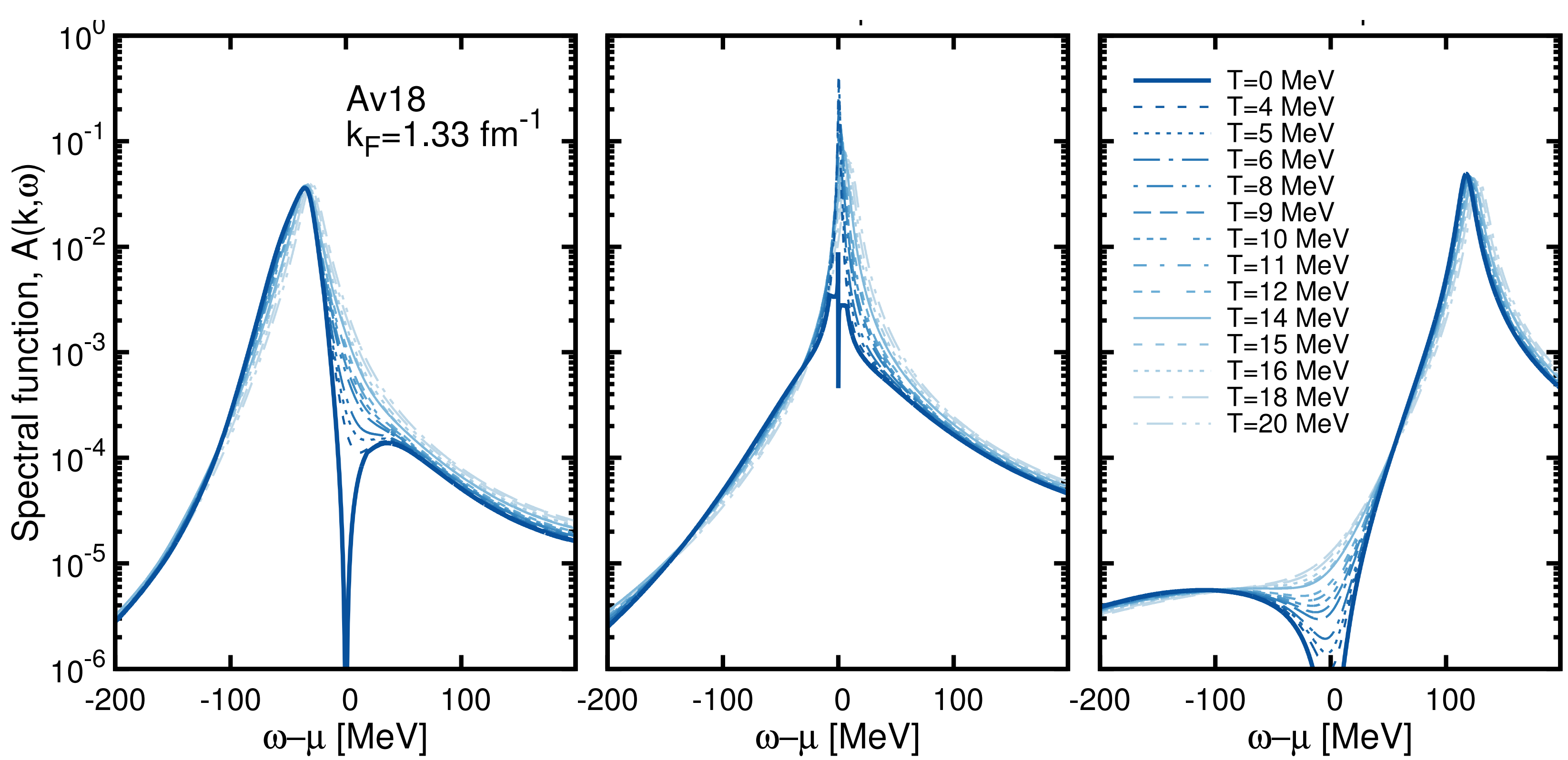


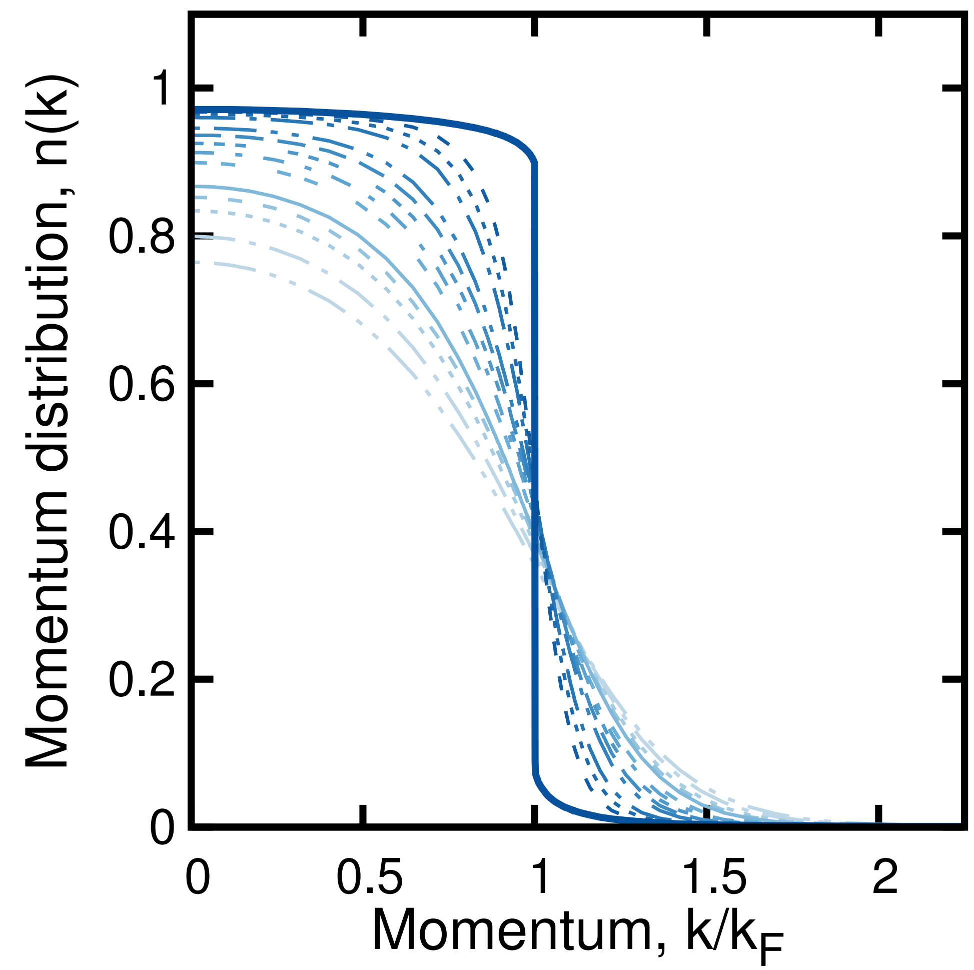
Self-energy, spectral function & thermodynamics

T=0 extrapolations





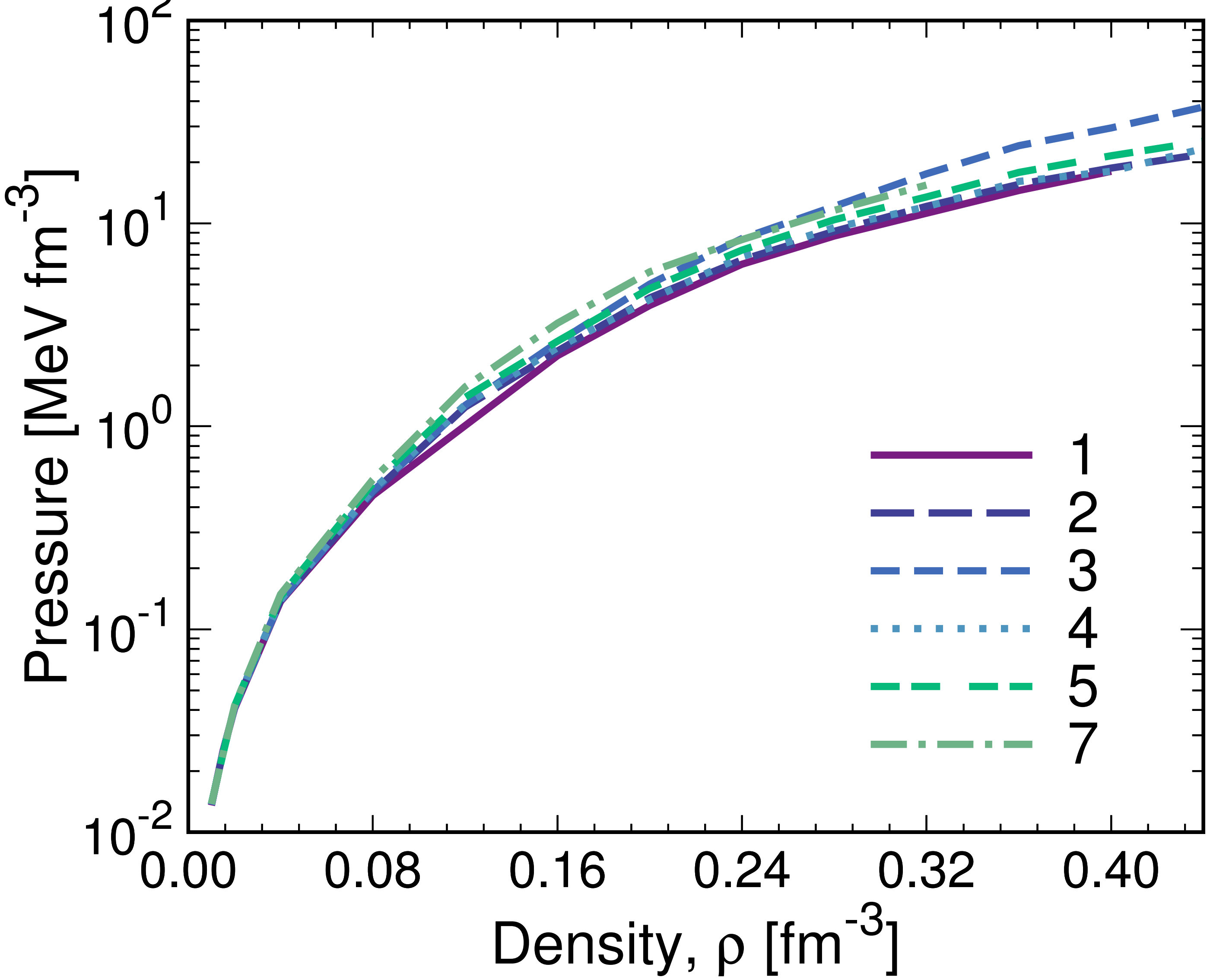




- **Extrapolate** $\Sigma(k,\omega;T)$ (13 Gb worth data)
- **Constraining** with **macroscopic** properties

Neutron matter

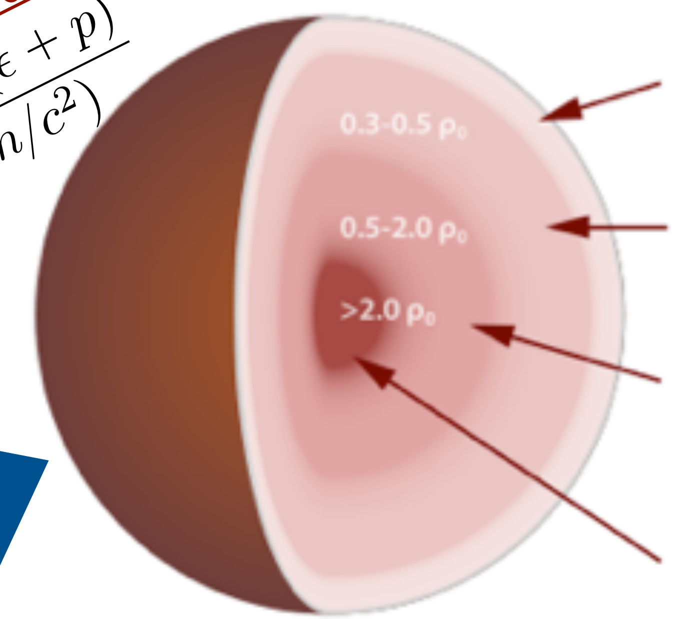
T=0 EoS for neutron matter: SRG

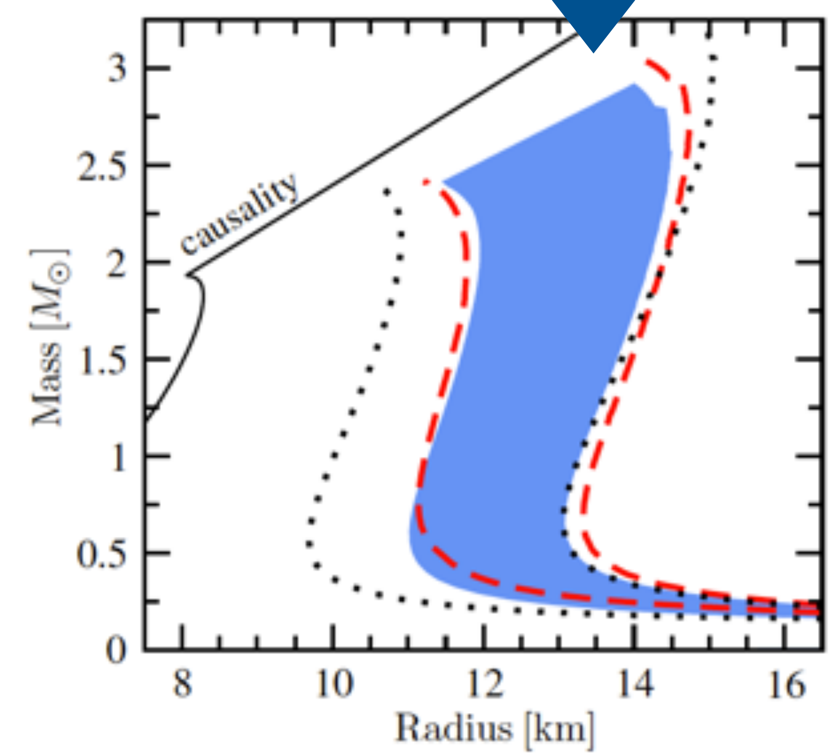


Tolman-Oppenheimer-Volkov

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m + 4\pi p r^3)(\epsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = \frac{4\pi}{c^2}\epsilon r^2$$





Hebeler, Lattimer, Pethick, Schwenk
ApJ **773** 11 (2013)

- Error band from fits in ChPT c_1, c_3 parameters
- Finite temperature & higher densities available

Carbone, Polls & Rios, *PRC* **90** 054322 (2014)
A. Carbone, Drischler, Hebeler, Schwenk, arXiv:1608.05615

Finite T EoS for neutron matter

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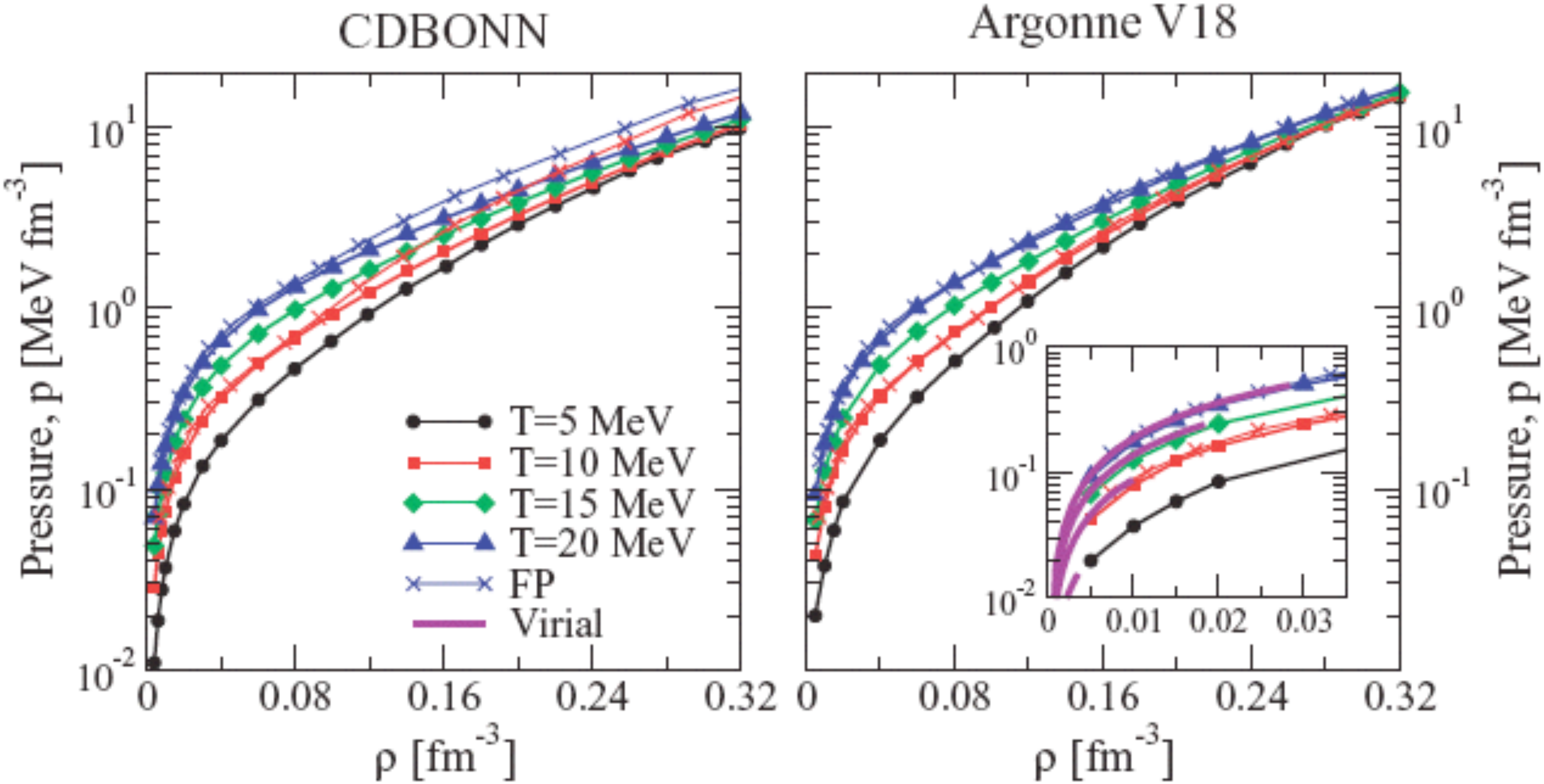
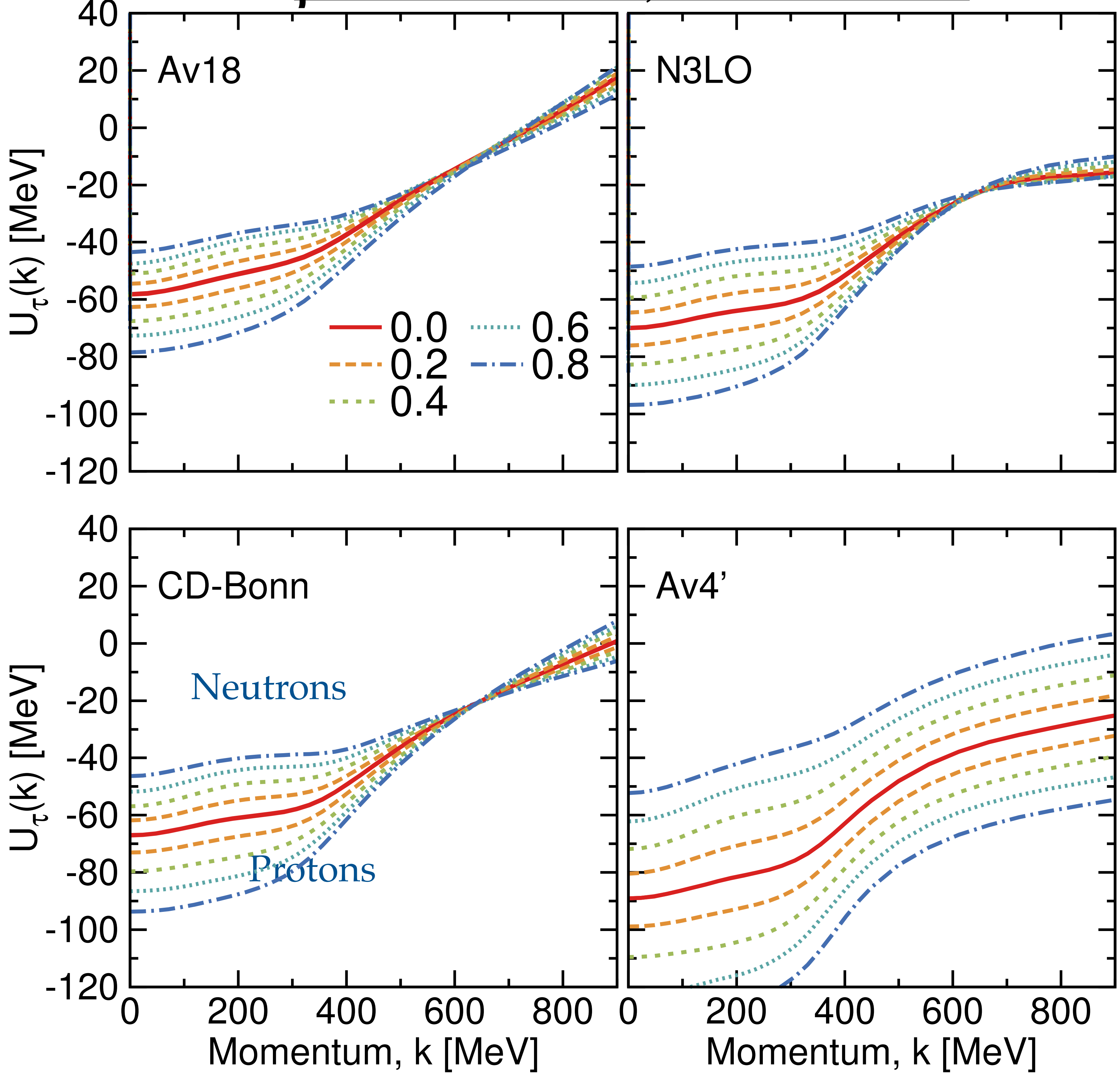


Rios, Polls & Vidana, Phys. Rev. C **79**, 025802 (2009)

- **Parameter-free** first principles calculation
- Reproduces **virial** at low density
- No need for **3NFs** (though we can treat them!)

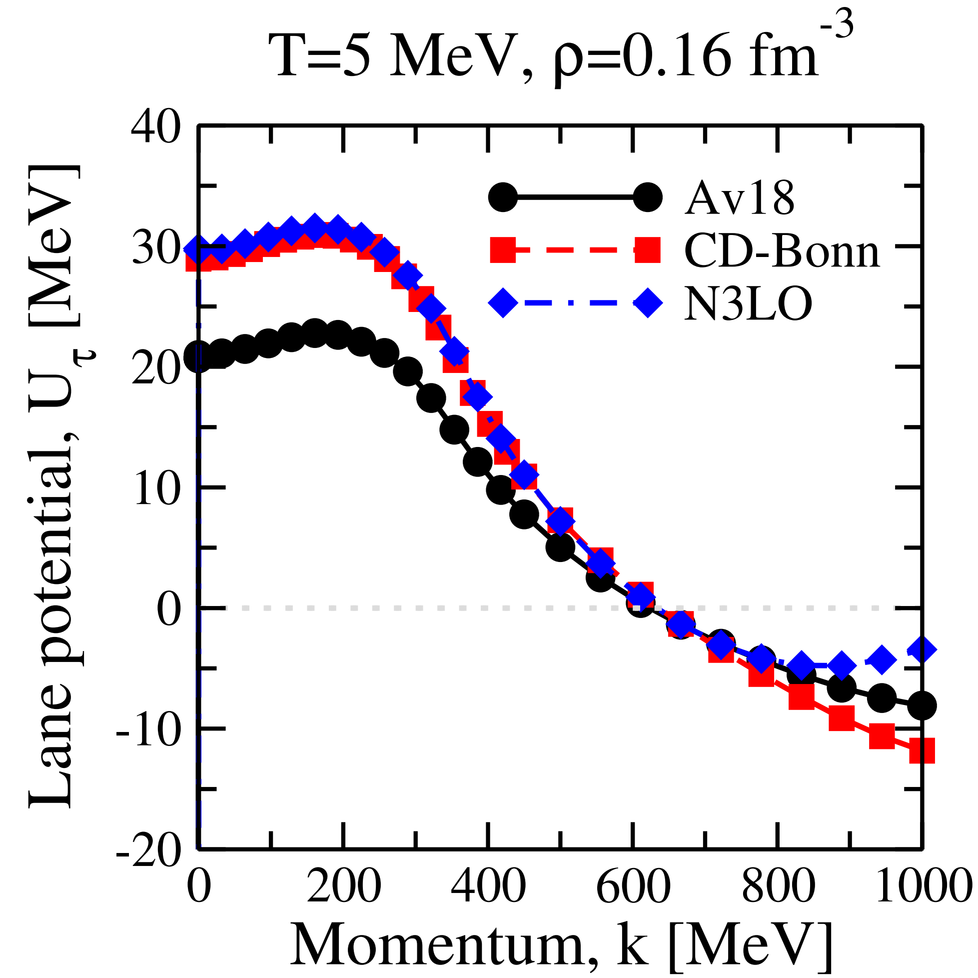
Single-particle potentials

ρ=0.16 fm^{-3}, T=5 MeV



s.f. peaks

$$\varepsilon_k^\tau = \frac{k^2}{2m_\tau} + \underbrace{\mathrm{Re}\Sigma^\tau(k,\omega=\varepsilon_k^\tau)}_{U^\tau(k)}$$



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Bardeen-Cooper-Schrieffer pairing

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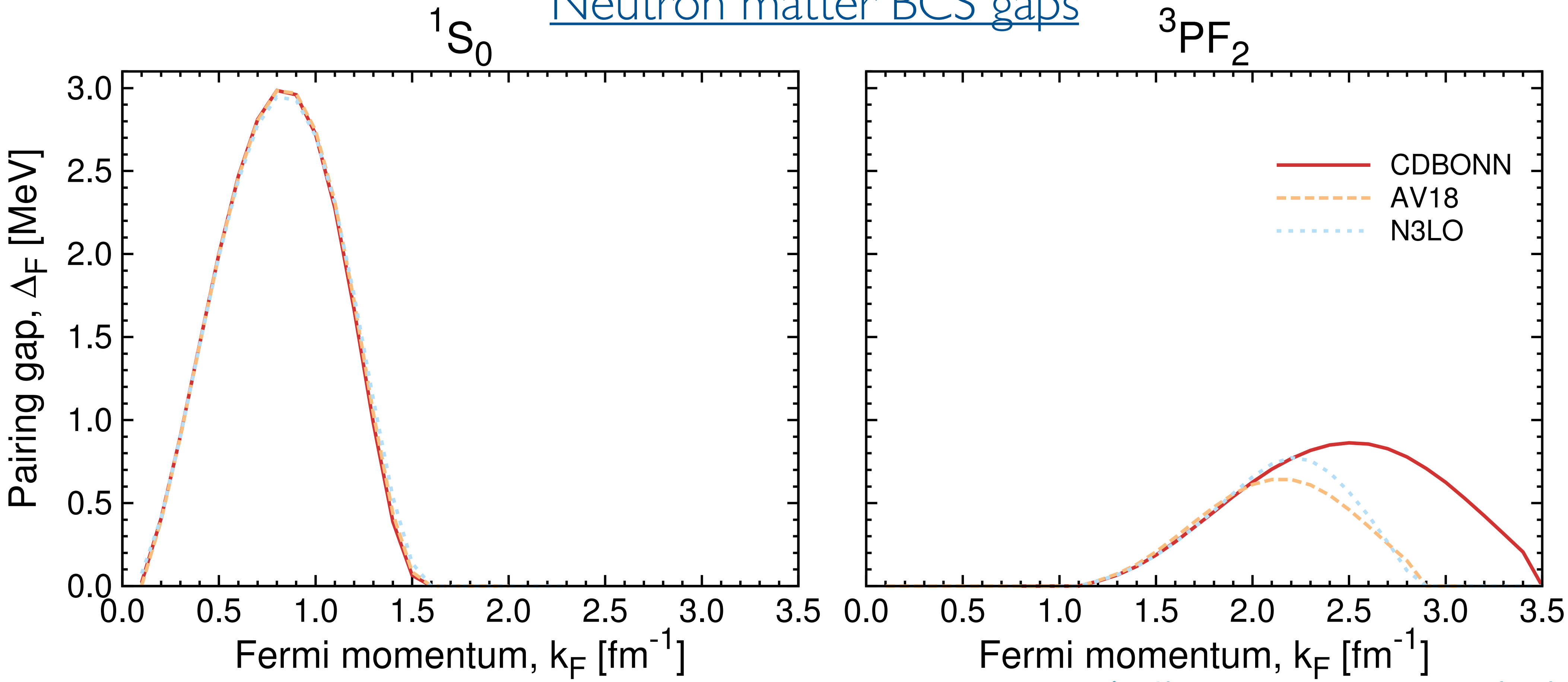
Neutron matter BCS gaps



Dean & Hjorth-Jensen, *Rev. Mod. Phys.* **75** 607 (2003)

BCS equation

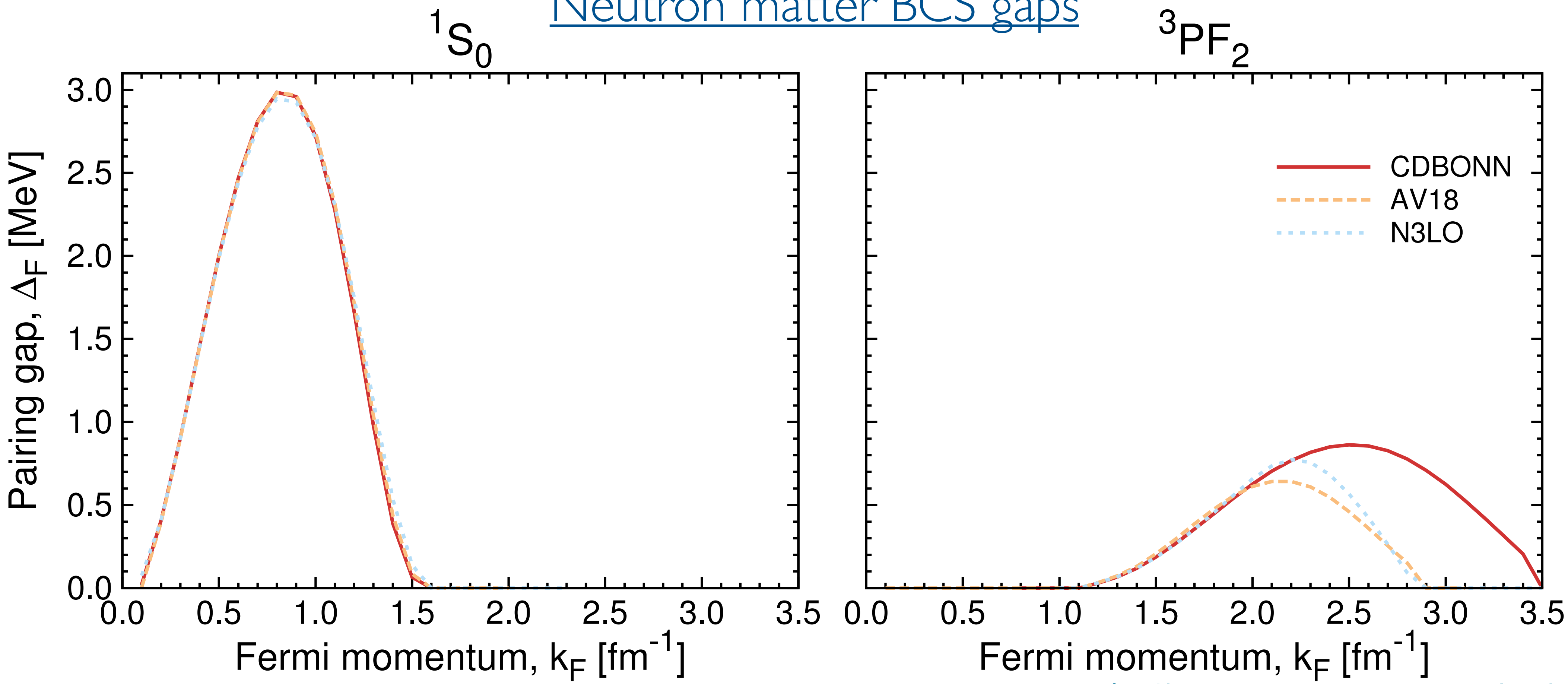
$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad \textbf{+} \quad \chi_k = \varepsilon_k - \mu$$

- Single-particle spectrum choice: $\varepsilon_k = \dfrac{k^2}{2m} + U(k) - \mu$
- Angular gap dependence: $|\Delta_k|^2 = \sum_L |\Delta_k^L|^2$

Maurizio, Holt & Finelli, *Phys. Rev. C* **90**, *044003* (2014)

Bardeen-Cooper-Schrieffer pairing

Neutron matter BCS gaps



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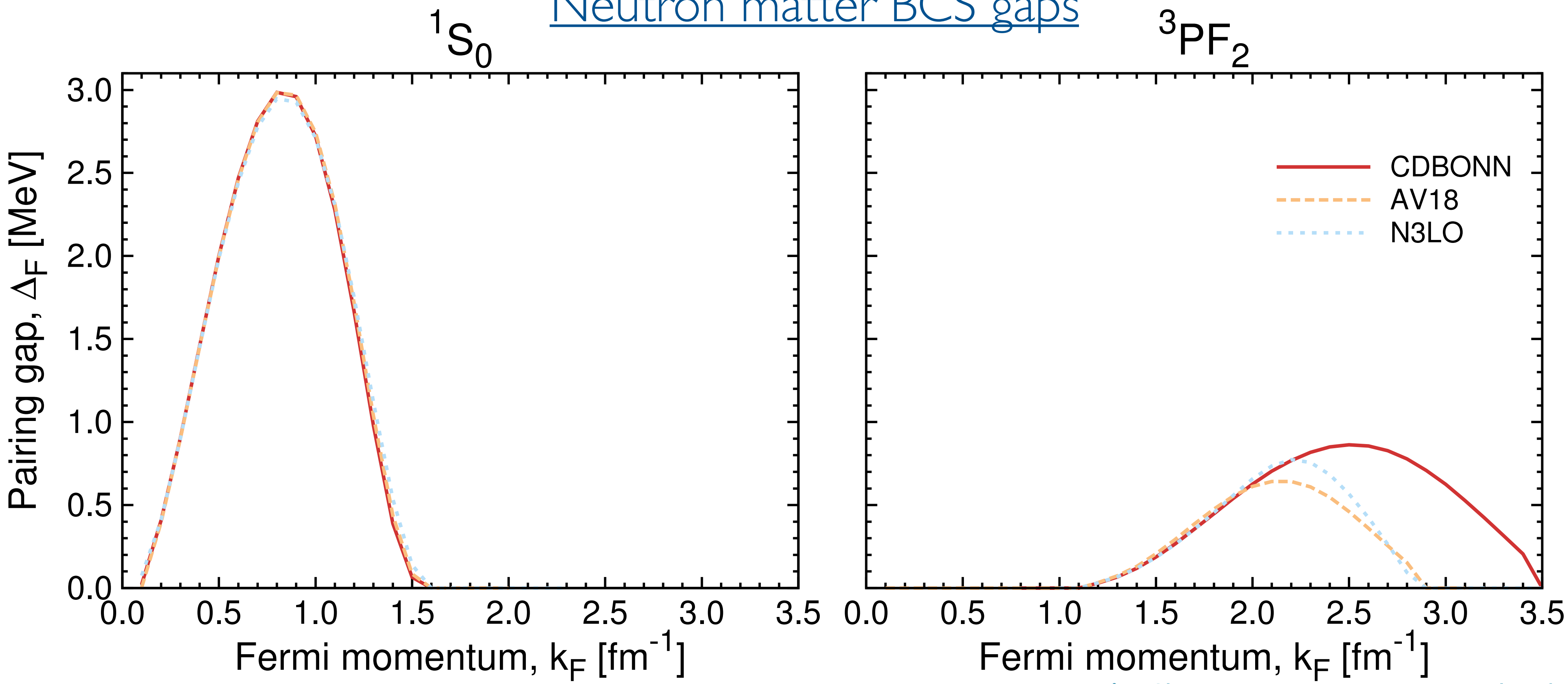

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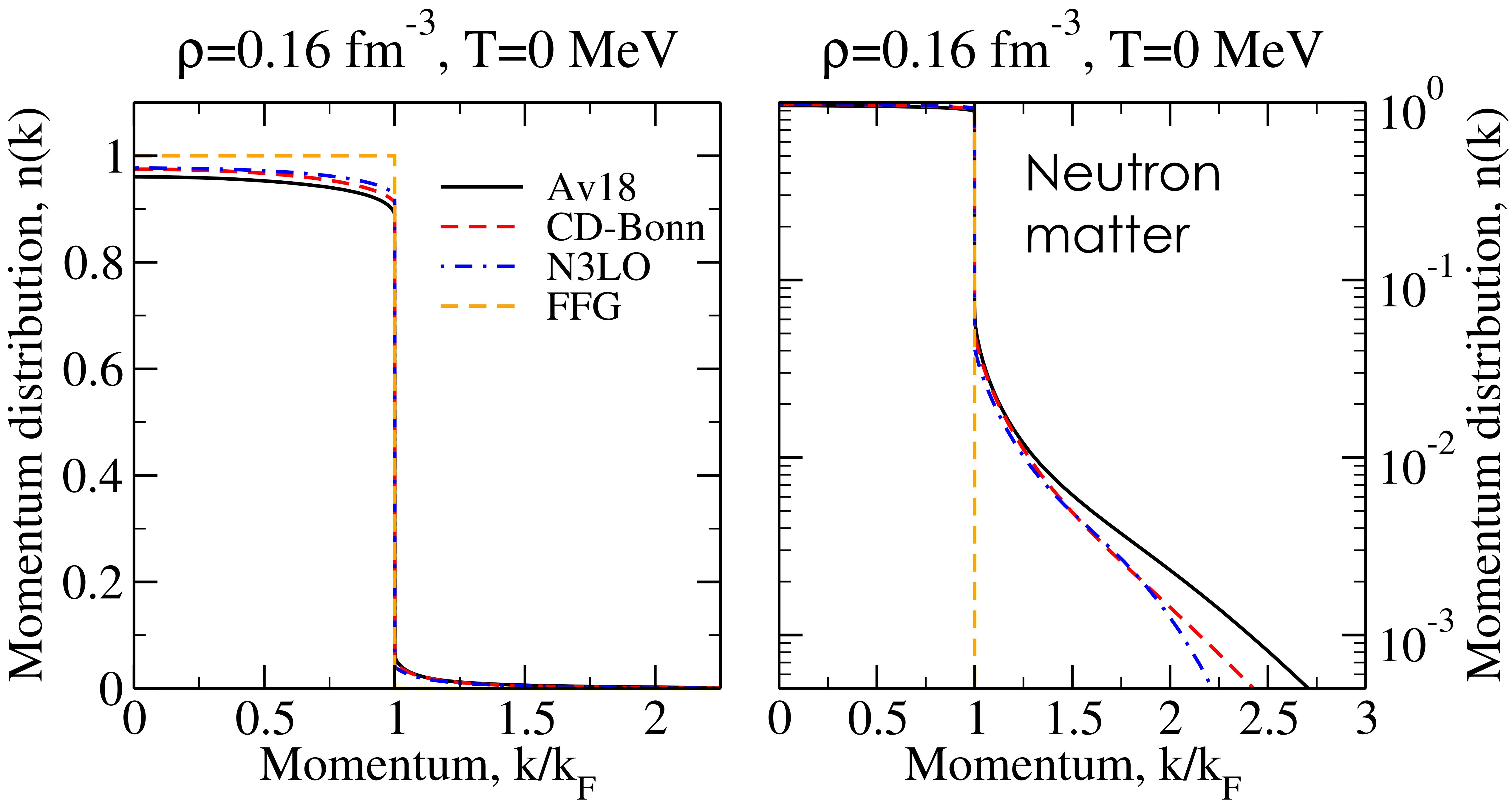
Maurizio, Holt & Finelli, *Phys. Rev. C* **90**, *044003* (2014)

Momentum distribution

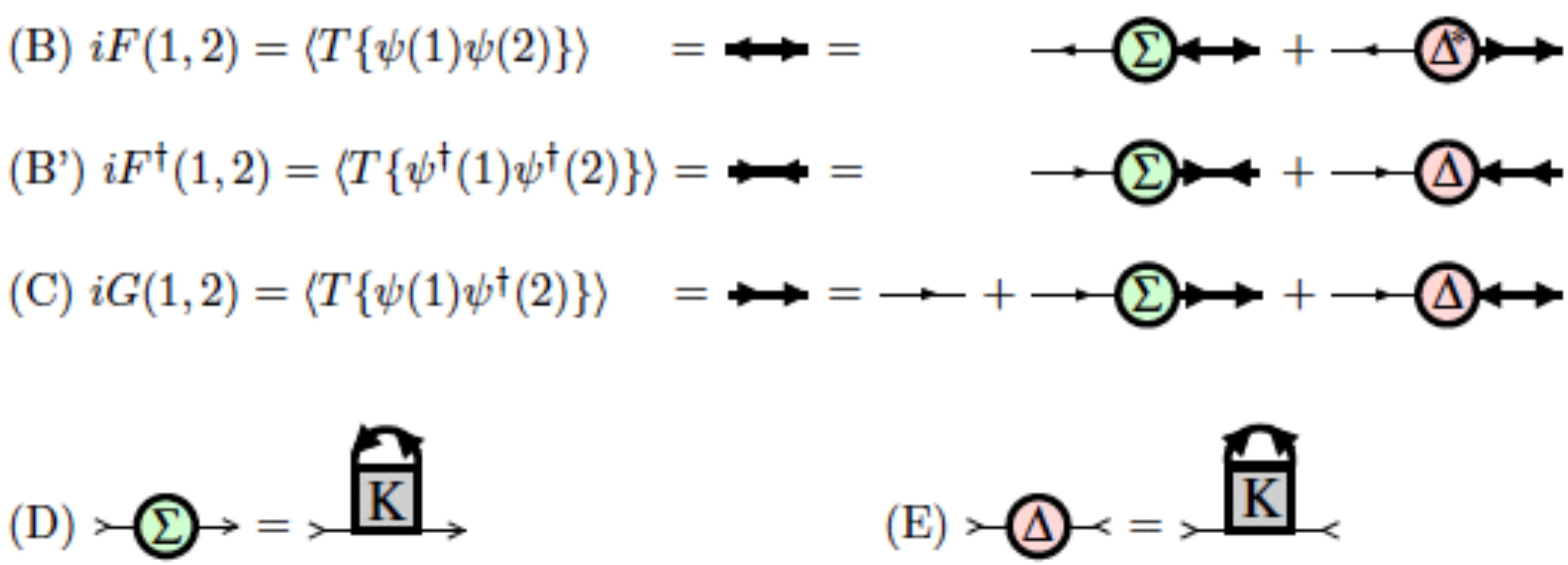
Single-particle occupation

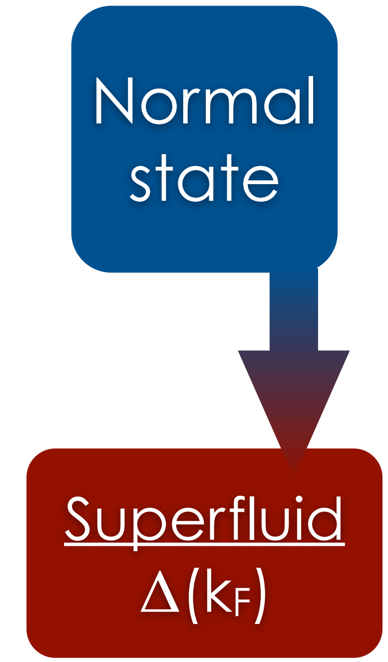
$$n(k) = \left\langle a_k^\dagger a_k \right\rangle \qquad \nu \int \frac{d^3k}{(2\pi)^3} n(k) = \rho$$



- SNM: 11-13% depletion at low k, population at high k
- Dependence on NN interaction under control
- PNM: 4-5% depletion at low k

Beyond BCS 101: SRC





BCS+SRC equation

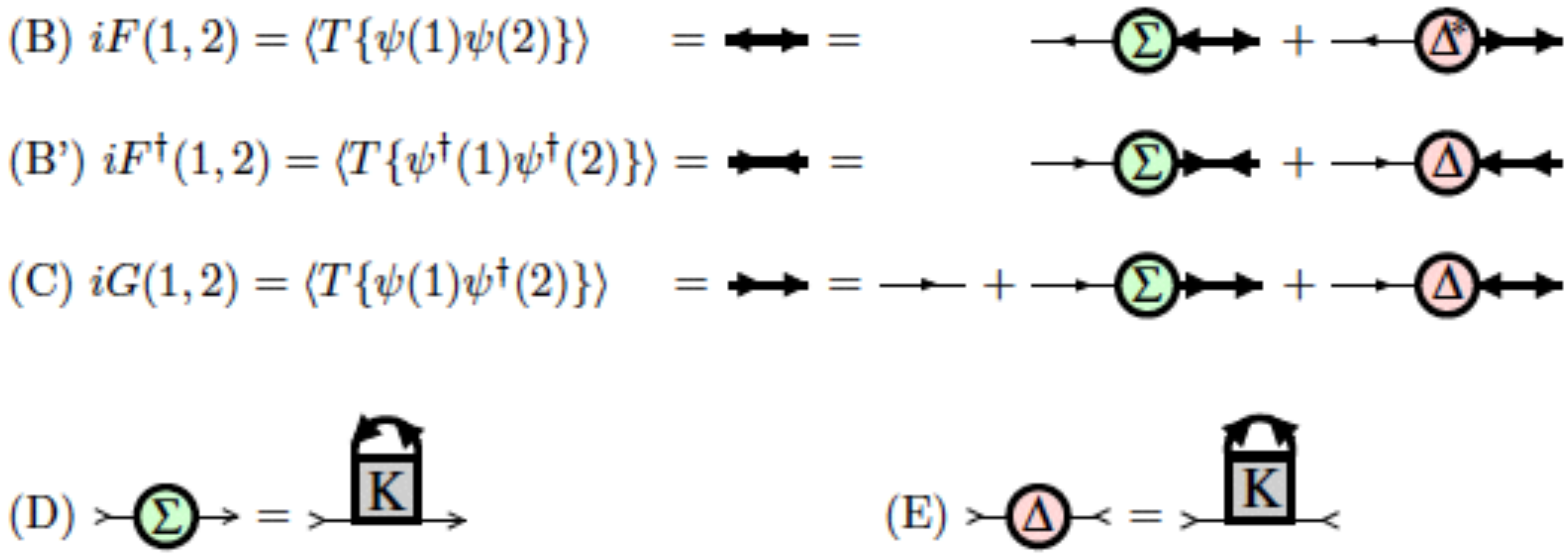
$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\bar{\chi}_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \; \mathbf{+}$$

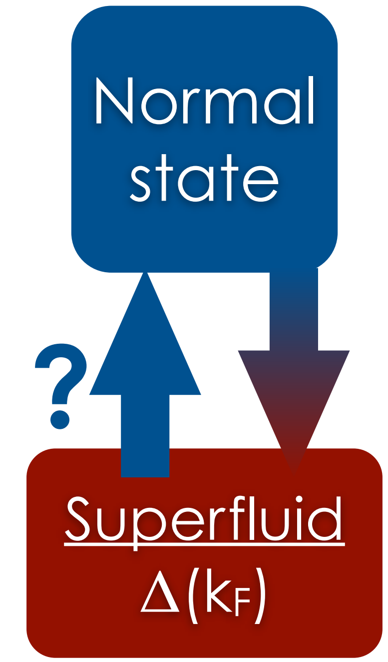
$$\frac{1}{2\chi_k} = \int_\omega \int_{\omega'} \frac{1 - f(\omega) - f(\omega')}{\omega + \omega'} A(k,\omega) A_s(k,\omega')$$

- **BCS** is *lowest order* in Gorkov Green's function expansion
- T-matrix can be extended to paired systems
- But full self-consistency is still missing

Bozek, *Phys. Rev. C* **62** 054316 (2000); Muether & Dickhoff, *Phys. Rev. C* **72** 054313 (2005)

Beyond BCS 101: SRC





BCS+SRC equation

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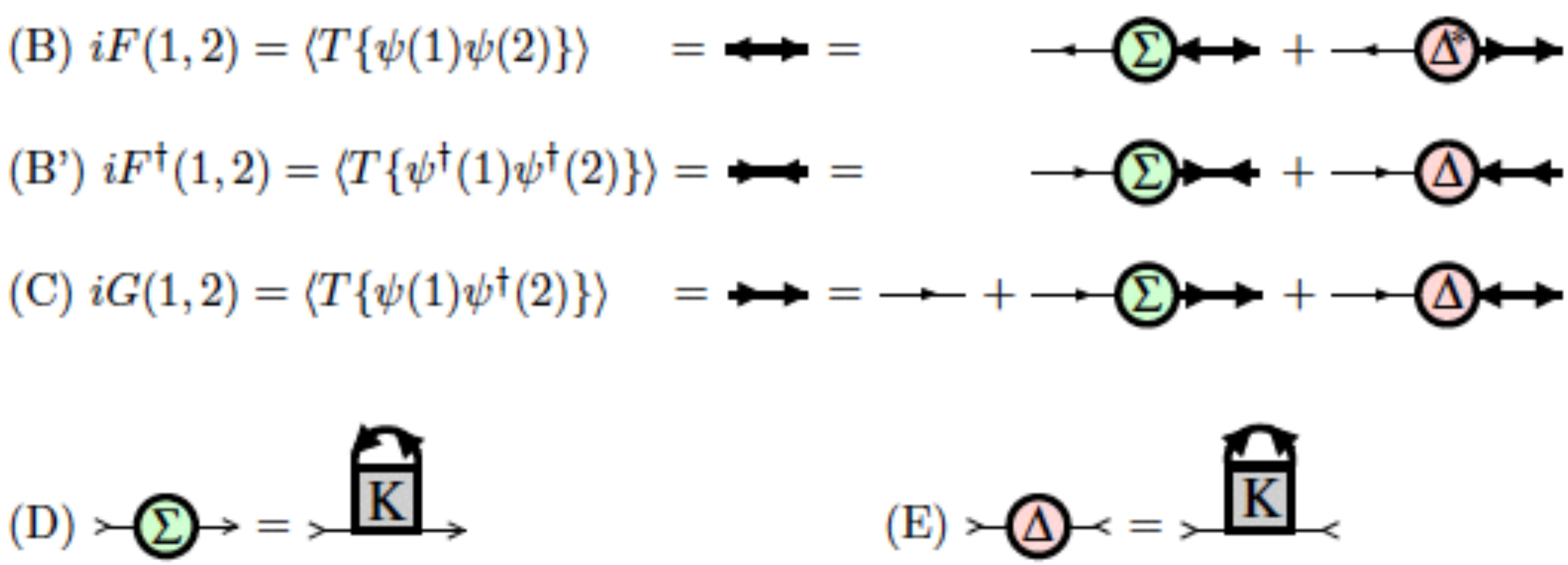

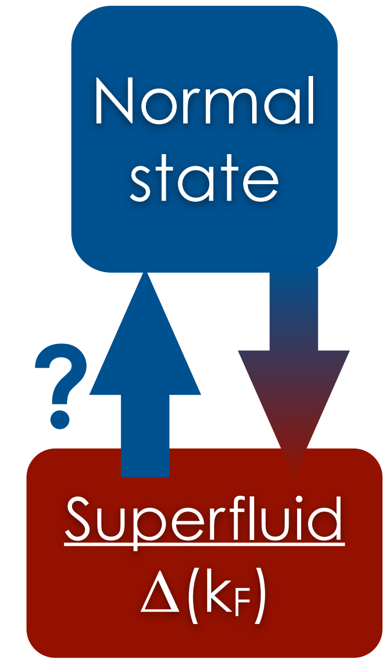

BCS+SRC equation

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- T-matrix can be extended to paired systems
- But full self-consistency is still missing

Bozek, *Phys. Rev. C* **62** 054316 (2000); Muether & Dickhoff, *Phys. Rev. C* **72** 054313 (2005)

Beyond BCS 201: LRC

UNIVERSITY OF SURREY

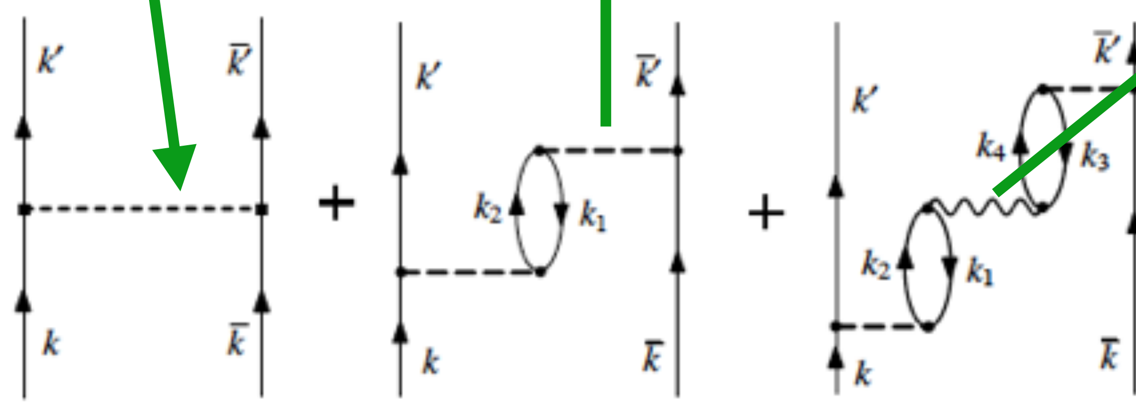
$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\boxed{\langle k | V_{nn}^{LL'} | k' \rangle}}{2\sqrt{\boxed{\chi_{k'}^2} + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$

?

ph recoupled G-matrix

Effective Landau parameters

$\mathcal{V}_{\text{pair}} =$



(a) (b) (c)

$$\langle 1\bar{1} | \mathcal{V} | 1\bar{1} \rangle = \frac{1}{4} \sum_{2,2'} \sum_{S,T} (-)^S (2S+1) \langle 12 | G_{ST}^{\text{ph}} | 1'2' \rangle_A \langle 2'\bar{1} | G_{ST}^{\text{ph}} | 2\bar{1}' \rangle_A \Lambda(22')$$

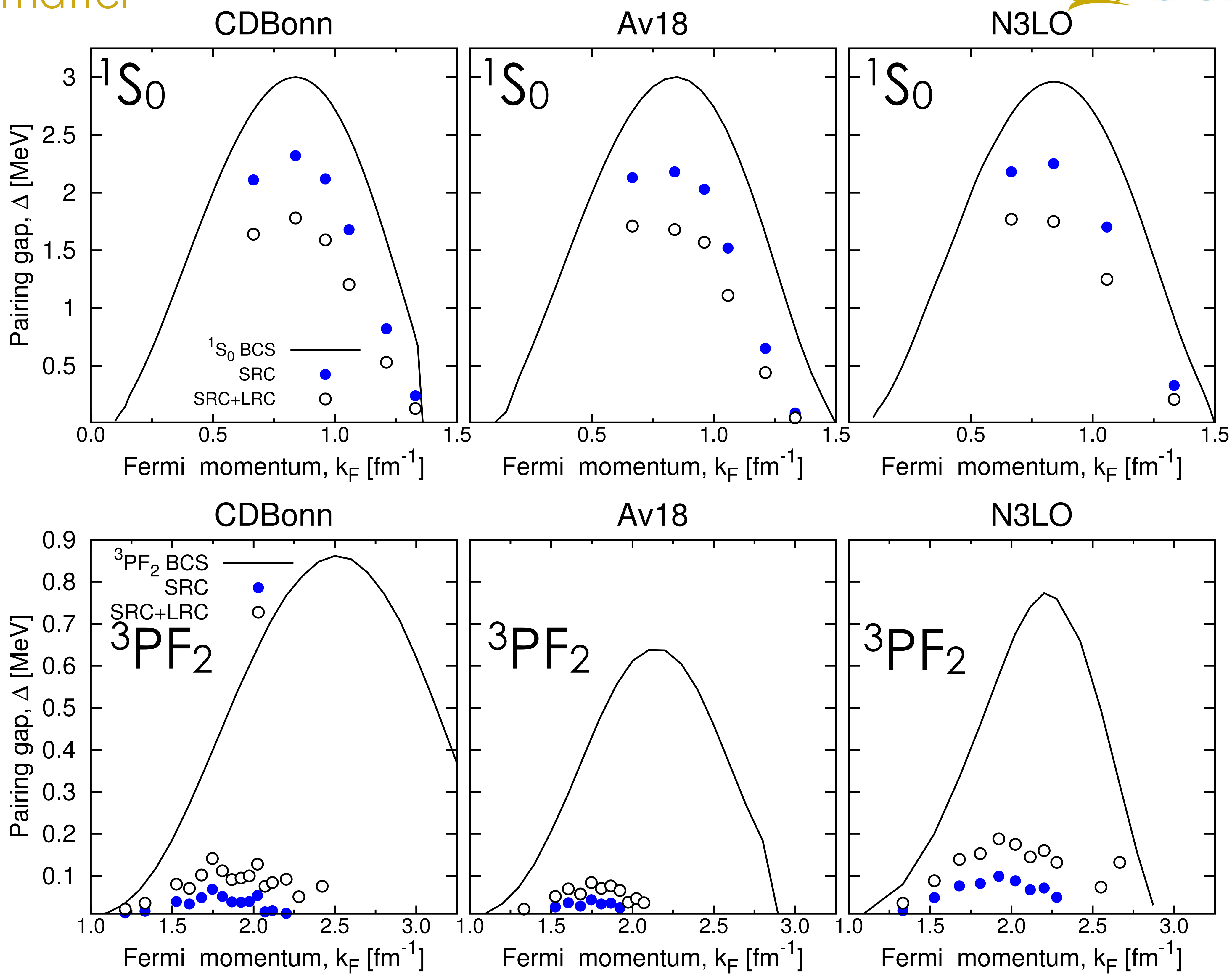
$$\Lambda_{ST}(q) = \frac{\Lambda_{ST}^0(q)}{1 - \Lambda_{ST}^0(q) \times F_{ST}}$$

- Bare NN potential only is not the only possible interaction
- Diagram (a): nuclear interaction
- Diagram (b): in-medium interaction, density and spin fluctuations
- Diagram (c): included by Landau parameters

Cao, Lombardo & Schuck, *Phys Rev C* **74** 064301 (2006)

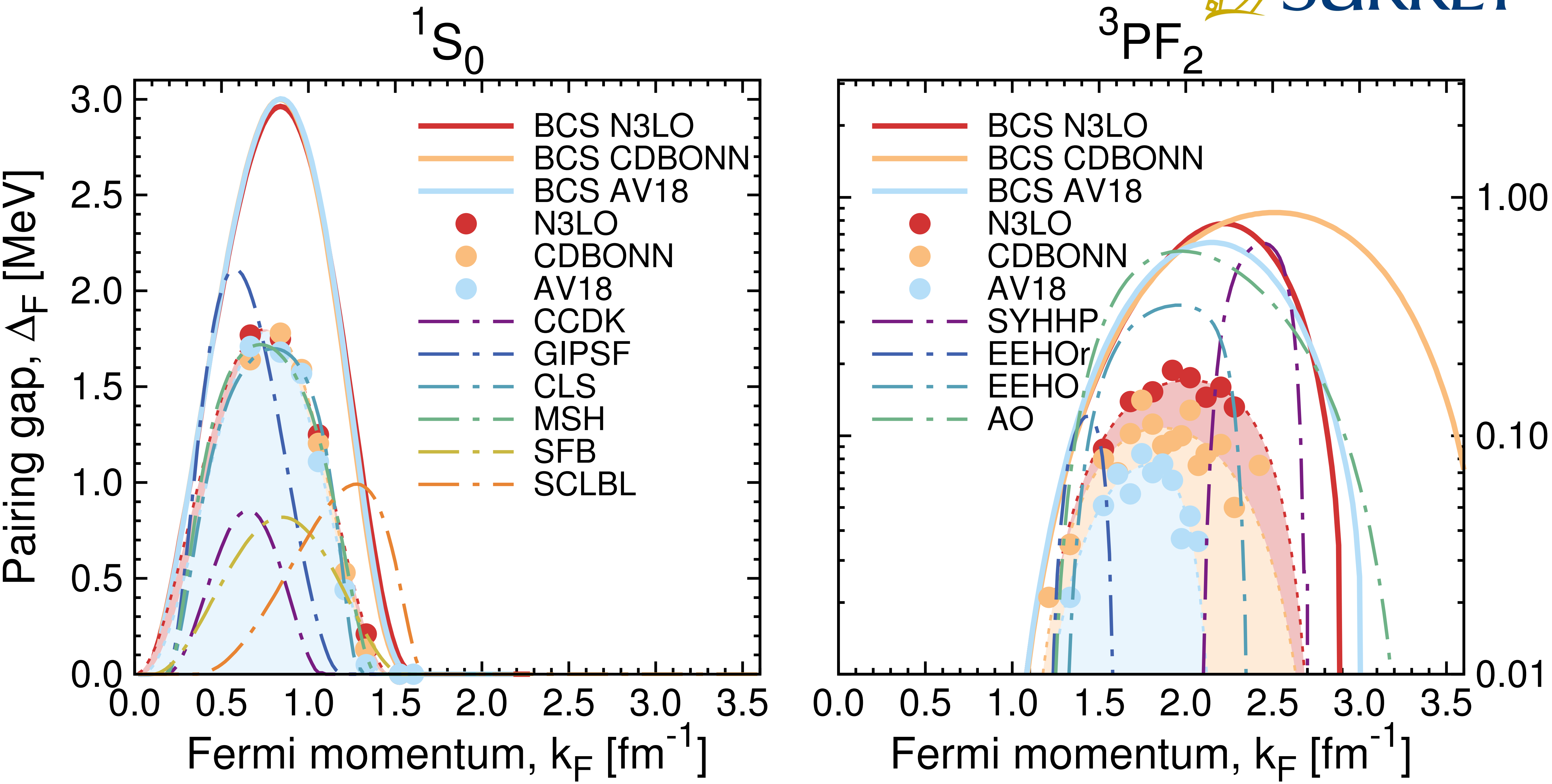
Beyond BCS 201: results $^1S_0$

Neutron matter



- LRC $^1S_0$ ($^3PF_2$) produces (anti-)screening

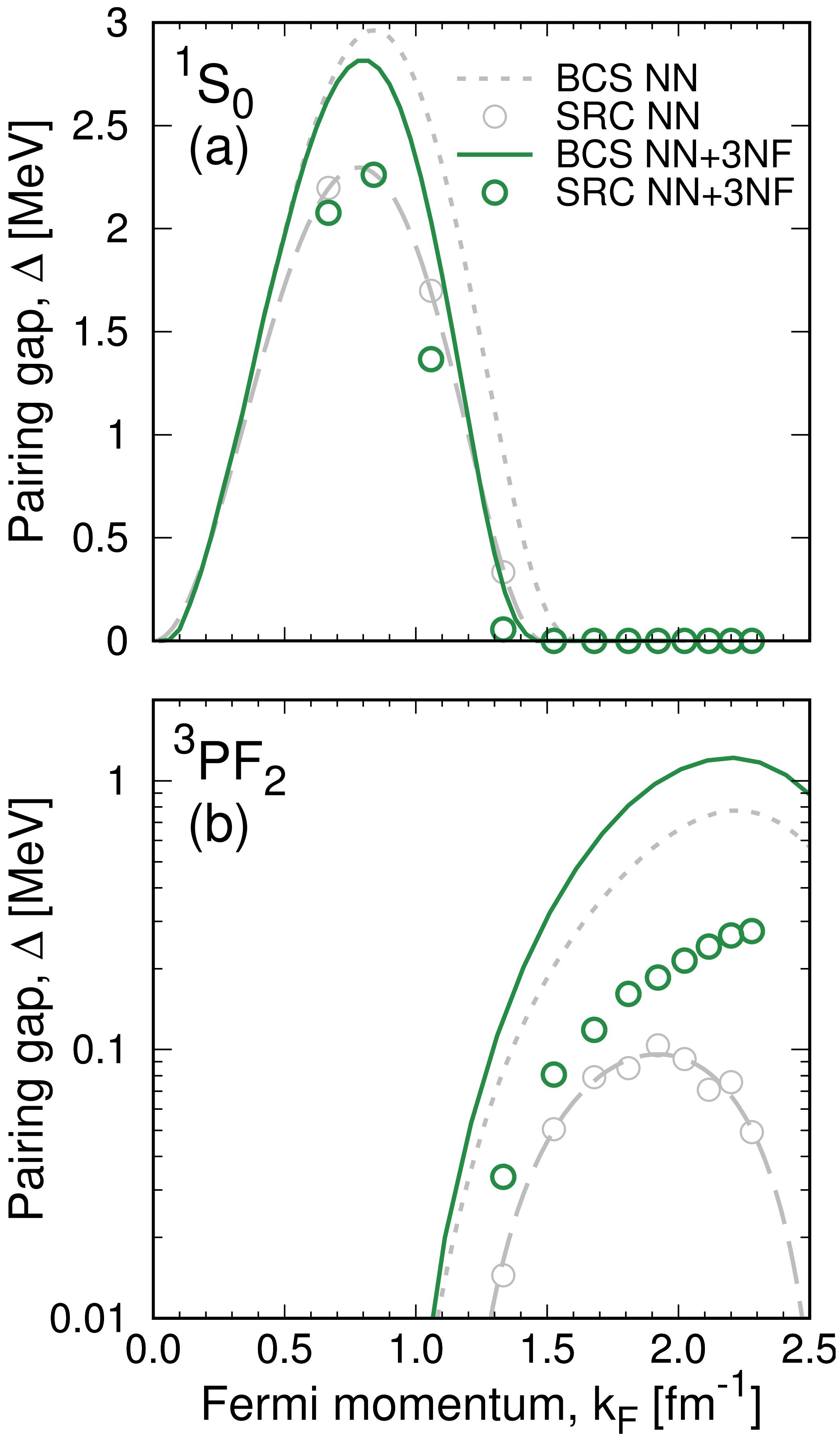
Beyond BCS pairing: overview



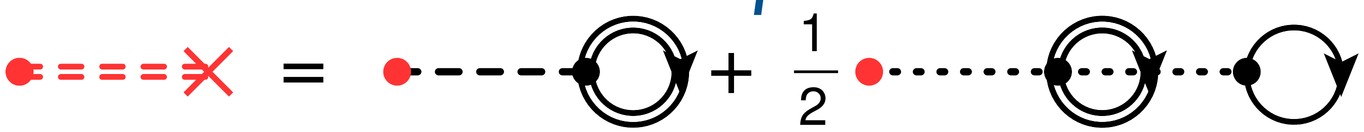
- **Singlet** channel under control in **astrophysical** situations
- **Triplet** is **different** from previous estimates
- 3NF effect **not** included in SRC, BCS indicates **small**

Ding, Rios, et al., Phys Rev C 94 025802 (2016)

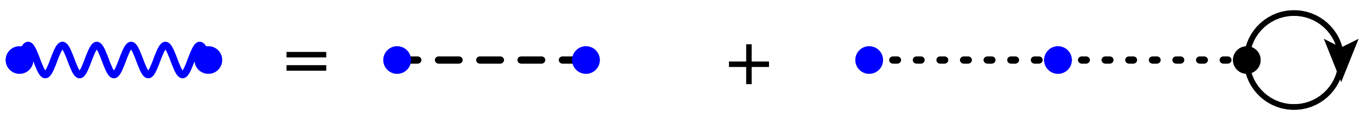
3BF effect: first estimate



Effective one-body force ⇒*spectrum*



Effective two-body force ⇒*NN forces*



- Singlet gap: 3NF **reduce** closure
- Triplet gap: 3NF **increase** gap
- **Model dependence** to be explored
- Use **SRG** for systematics

Collaborators

+A. Carbone

+D. Ding, W. H. Dickhoff

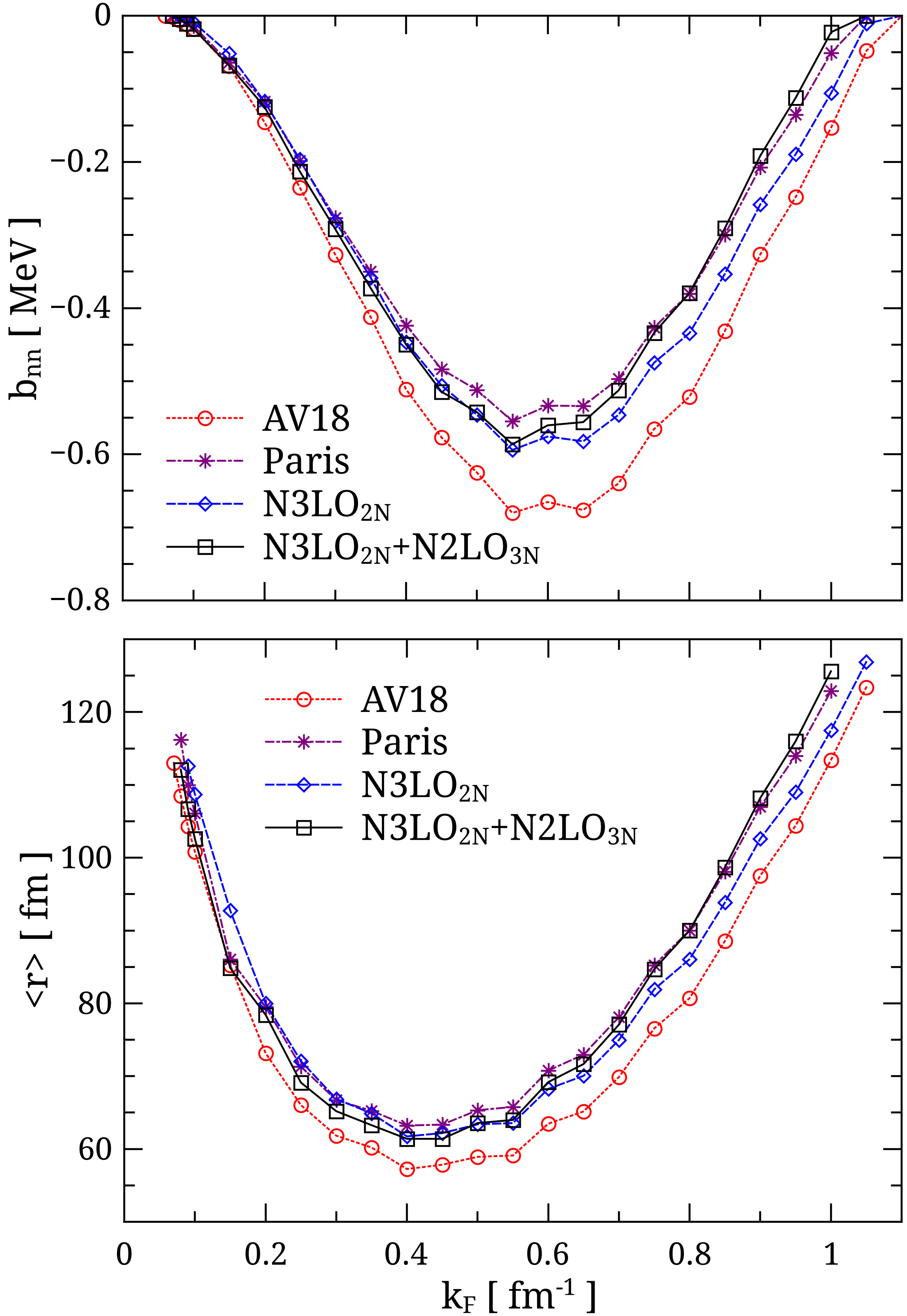
+A. Polls

+C. Barbieri

+V. Somà

+H. Arellano, F. Isaule

Dineutrons



BHF G-matrix

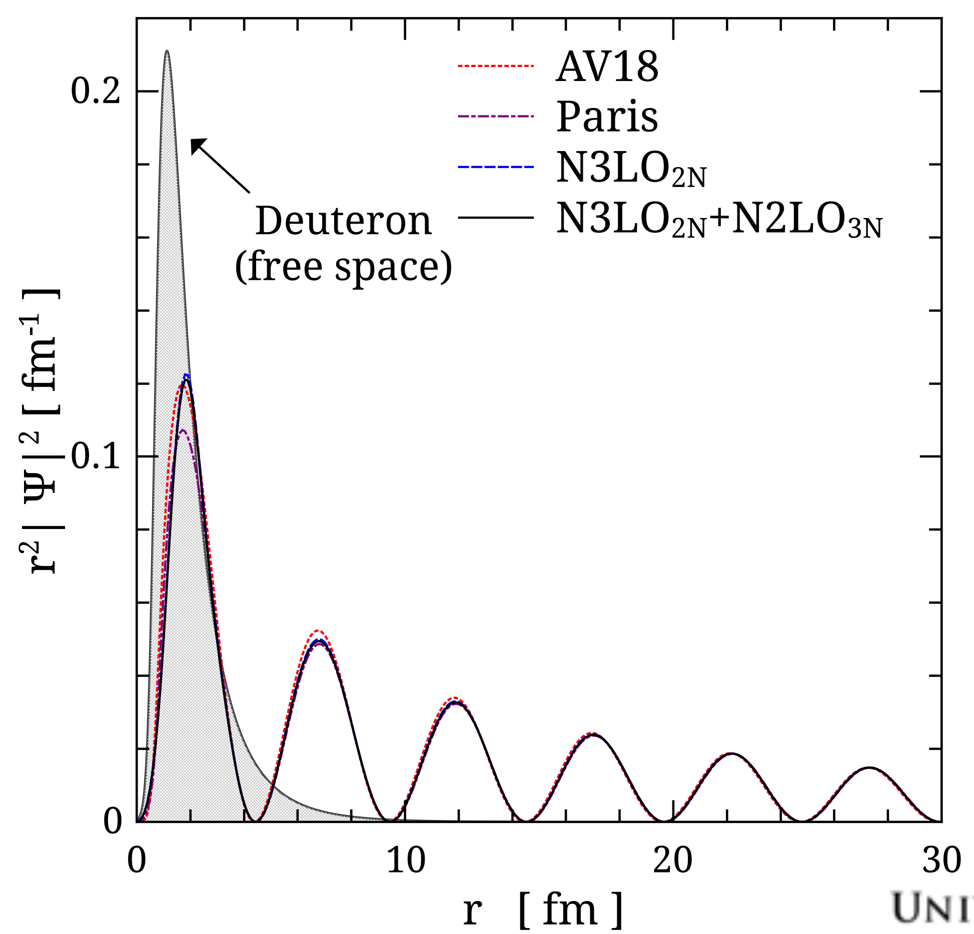
$$\mathcal{G}(\omega) = \mathcal{V} + \mathcal{V}Q(\omega)\mathcal{G}(\omega)$$

Poles

$$\det\left[1 - \mathcal{V}\mathcal{Q}(\omega_i)\right] = 0$$

$$b_{\mathrm{nn}} = \omega_i - \omega_{\mathrm{th}}$$

k_F=0.6 fm^{-1}



Isaule, Arellano, Rios, *Phys Rev C* ***94***, *034004 (2016); EPJA* ***52***, *299 (2016)*.

Conclusions

- Towards **consistent** models of NS core
- **Talk** to us if you need **quantitative** predictions!

- Ab initio nuclear **theory** needed to treat **correlations**
- **Different** NN forces give systematic errors

- Challenges ahead:
  - **Pairing** in isospin **asymmetric** matter
  - **Consistent** treatment of **cooling**, **glitch** & **EoS**
  - Improvements of **many-body theory**

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