Nuclear Data Needs for the Equation of State in Core Collapse Supernovae

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Equation of State Conditions in Core Collapse Supernovae

- density ρ : 0 to 10^{15} g cm^{-3}
- temperature T: 0 to 30 MeV
- electron fraction Y_e : ~ 0.5 to ~ 0

For densities below 10^7 g cm^{-3}, large Y_e, low T: Known nuclear masses with Nuclear Statistical Equilibrium (NSE) with Coulomb corrections, or network

Focus here will be for densities $> 10^7$ g cm^{-3}:

- extrapolation of nuclear masses to lower Y_e, higher A
- inclusion of nuclei - external $n - p$ gas interactions
- inclusion of nuclear excited states
- phase transition to bulk nucleon matter around $\rho_s/3 - \rho_s/2$, $n_s = 0.16$ fm^{-3}, $\rho_s = 2.7 \times 10^{14}$ g cm^{-3}
- uncertain properties of underlying nuclear force (K, K', S_v, $S_v(n)$, S_s)

Primary assumption: Single nucleus approximation

Distribution of nuclei replaced by the most energetically favored nucleus. Can be demonstrated to introduce error in pressure $\simeq nT/A$, chemical potentials $\simeq T/A$ which are negligible compared to errors introduced by ambiguities in nuclear force parameters.

Leptons can be treated separately from baryons

Main Classes of Nucleon Force Models

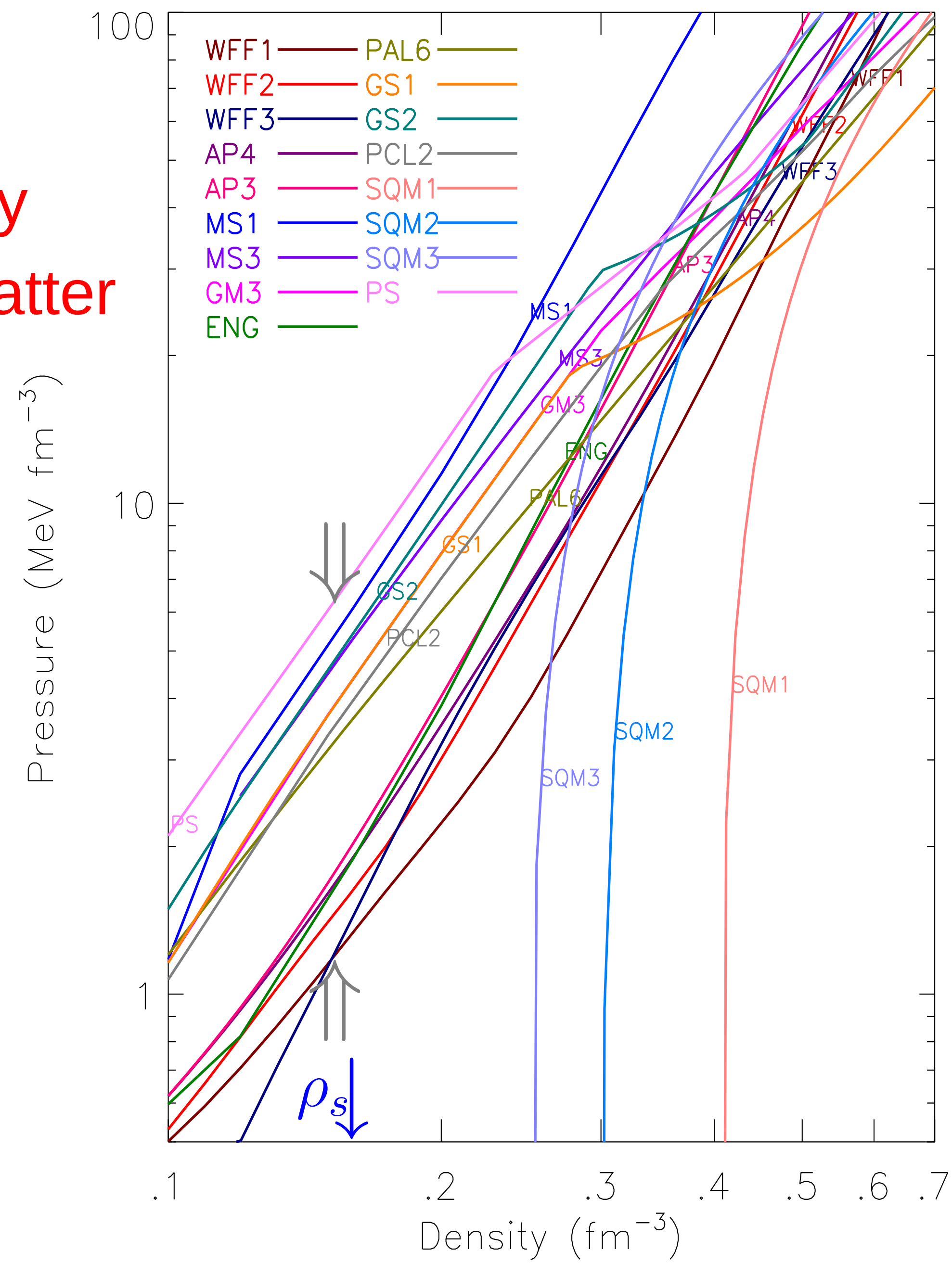
- ## Non-relativistic potential models
  - Momentum- and density-dependent potential
  - Power-series density expansion
  - Density-dependent effective nucleon masses
  - Relatively slowly varying $S_v(n)$, smaller neutron star radii (but adjustable)
  - Can become acausal
  - Can be constrained to fit low-density matter properties and hypernetted chain calculations
- ## Relativistic field-theoretical models
  - Interactions mediated by bosons (ω, σ, ρ)
  - Implicitly causal
  - Generally have linearly increasing $S_v(n)$, larger radii (also adjustable)
  - Not as easily constrained to fit low-density matter properties and hypernetted chain calculations
- ## High-density 'exotica'
  - Strangeness in form of hyperons, kaon/pion condensates, deconfined quark matter; many fewer laboratory constraints
  - Possibility of stable strange quark matter at zero pressure (Witten)
  - Not important to supernova models (suppressed by large μ_e), important in protoneutron stars

Neutron Star Matter Pressure

Extrapolation in symmetry
to nearly pure neutron matter

Wide variation:

$$1.2 < \frac{P(\rho_s)}{\mathrm{MeV\ \ fm^{-3}}} < 7$$



Major existing approaches for $\rho < \rho_s/2$ *tables*

- Liquid droplet models
  - Baym, Bethe & Pethick 1971
  - Lattimer et al. 1985; Lattimer & Swesty 1990
    Non-relativistic potential, surface energy from semi-infinite plane-parallel calculation, Coulomb energy from droplet model
- Finite-temperature Thomas-Fermi unit cell calculations
  - Oyamatsu 1993, Shen et al. 1998
    Relativistic mean field theory, suface and Coulomb energies from density profile optimazations
- Finite-temperature Hartree-Fock unit cell calculations
  - Bonche & Vautherin 1980, Wolff & Hilledbrandt (ca. 1985)

Lattimer & Swesty 1991, *NPA* **535**, *331*

- Based on Lattimer, Pethick, Ravenhall & Lamb [NPA **432**, 646, (1985)] liquid droplet model merged with bulk equilibrium
- Free energy density is minimized

$$F = un_i[f_i + f_{surf} + f_{Coul} + f_{trans}] + (1 - u - n_N n_\alpha v_\alpha)f_o + n_\alpha f_\alpha(1 - u)$$

$$\mu_{no} = \mu_{ni} + \Delta_n, \quad \mu_{po} = \mu_{pi} + \Delta_p, \quad P_o = P_i + \Delta_P, \quad f_{surf} = 2f_{Coul}$$

- f_i, f_o from non-relativistic potential (Skyrme-like) model
- f_{surf} from semi-infinite plane-parallel calculations using f_i, f_o and gradient contributions, but ignoring Coulomb effects
- f_{Coul}, f_{trans} from liquid-drop model including lattice effects
- f_α for Maxwell-Boltzmann particles to represent "light" nuclei
- uniform densities inside and outside nucleus
- L-S ignores "neutron skin" and nucleon effective masses to simplify minimization although LPRL includes these
- Phase transition to uniform matter treated with Maxwell construction
- L-S includes nuclear shape (i.e., nuclear pasta) variations
- LPRL assumed SI′ model; L-S contains arbitrary parameters to match input incompressibility and bulk and surface symmetry energies

Shen, Toki, Oyamatsu & Sumiyoshi 1998,

PTP **100**, *1013*

- Based on Oyamatsu [NPA **561**, 431 (1993)]
- Thomas-Fermi spherical cell model
- Uniform matter energy from relativistic field-theoretical (RFT) model with σ, ω, ρ mesons
- Ad-hoc density-gradient contribution
- Coulomb contributions for non-uniform matter included
- Light nuclei represented as Maxwell-Boltzmann particles
- Energy minimization assuming parametrized Fermi-like density profiles
- Phase transition to uniform matter ignored
- Nuclear shape variations ignored
- Tables for one set of RFT parameters

Current problems

L-S

- α-particle binding energy error
- Unity effective masses underestimate nuclear specific heats
- Doesn't work for extremely small temperatures and proton fractions and some points near critical temperature/density (convergence issues)
- Ignores neutron skin and Coulomb corrections to surface energy
- Extension to other nuclear models restricted by physical labor involved in computation of phase boundaries

Shen et al.

- table relatively sparse
- Not possible to implement "thermodynamically consistent" table generation scheme (Swesty & Timmes)
- Tables for alternate incompressibility and symmetry properties not available
- Inconsistent surface energies (possibly reflected in anomalously small neutron skin thicknesses)
- Does not consider aspherical geometries

Current improvements (L-S)

- α-particle binding energy error corrected (comparison to NSE calculations at low density satisfactory [Hix])
- Nuclear force generalized for arbitrary effective masses, both NRP and RFT models utilized
- Re-introduction of neutron skin
- Energy minimization without algebraic substitutions (i.e., 5 simultaneous equations rather than 3)
- Automatic table generation – no convergence issues
  - Two finely gridded tables (one assuming nuclei, the other uniform matter) used to generate phase boundaries, identify table points within Maxwell construction and replace appropriate values

The Next Generation:

Finite-Range Thomas-Fermi Model

Based on Thomas-Fermi model of Myers & Swiatecki [AP, **204**, 401 (1990)], but extended to finite temperature

$$W = -\frac{1}{h^3}\int d^3r_1 \int d^3r_2 f\left(\frac{r_{12}}{a}\right) \times$$

$$\sum_{t1,t2,t'2=n,p}\left[\int\int C_L f_{t1} f_{t2} d^3p_{t1} d^3p_{t2} + \int\int C_U f_{t1} f_{t'2} d^3p_{t1} d^3p_{t'2}\right]$$

$$C_{(L,U)} \propto \alpha_{(L,U)} - \beta_{(L,U)}(\frac{p_{12}}{P_o})^2 - \sigma_{(L,U)}(\frac{2\bar{\rho}}{\rho_o})^{2/3}$$

$$f\left(\frac{r_{12}}{a}\right) = \frac{1}{4\pi r_{12} a^2} e^{-r_{12}/a}, \quad r_{12} = |\vec{r_1} - \vec{r_2}|, \quad \int d^3r_2 f\left(\frac{r_{12}}{a}\right) = 1$$

In contrast, Skyrme force Hamiltonian:

$$\mathcal{H}_{Skyrme} = \mathcal{H}_{uniform}(n,\tau) + \sum_{i,j=n,p} Q_{ij} \nabla n_i \cdot \nabla n_j$$

Energy minimization – Euler equations

FRTF

Integral equations:

$$\mu_n(r) = \text{ constant}, \qquad \mu_p(r) = \text{ constant} \tag{1}$$

NRP

Differential equations:

$$\sum_{j=n,p} Q_{ij} \nabla^2 n_j = \frac{\partial \mathcal{H}_{uniform}}{\partial n_i} - \mu_{i0} \tag{2}$$

FRTF Example