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The equation of state (EoS) from the user perspective

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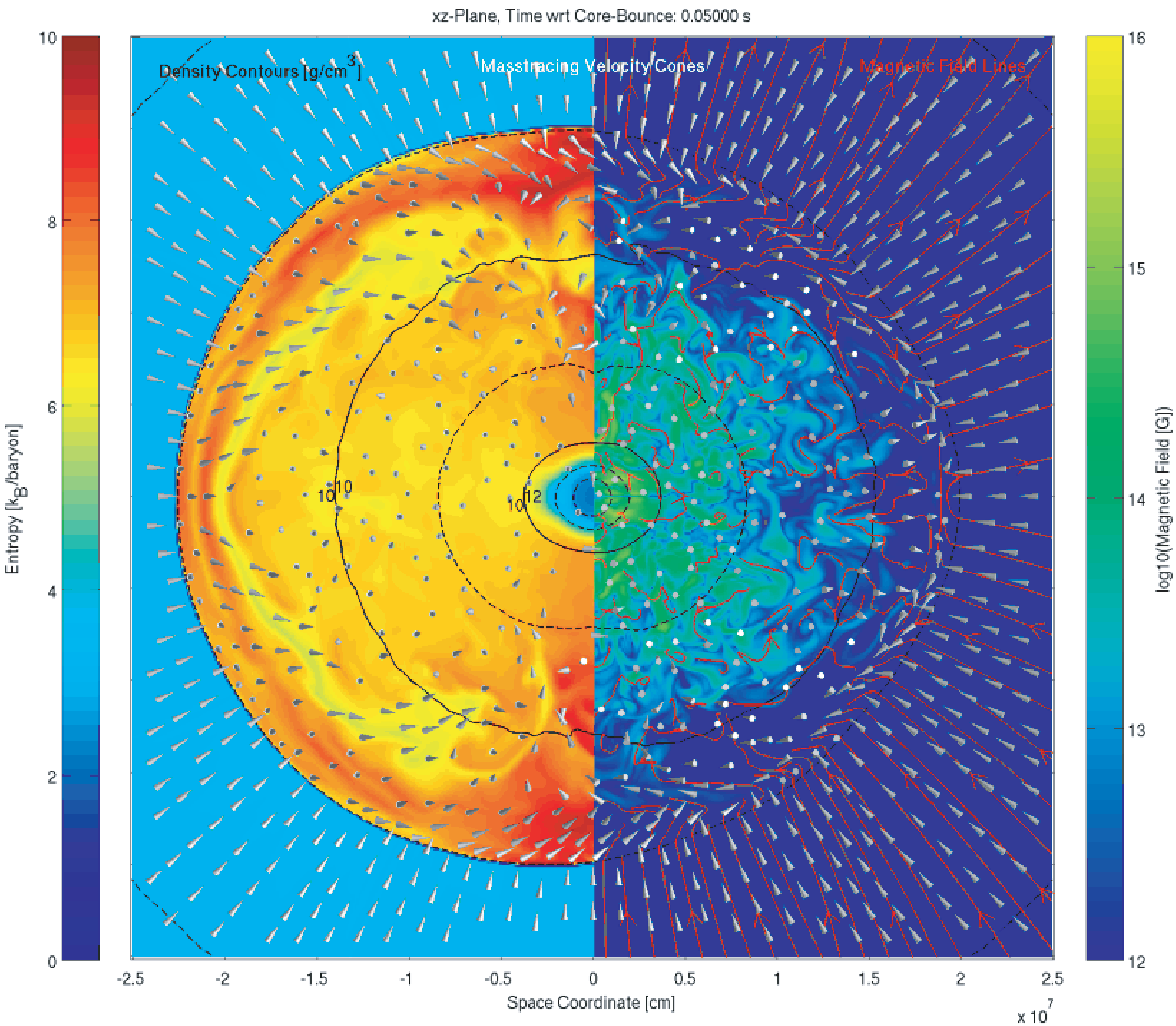
1) Basic physical/astronomical question:	• Why do we need an EoS and which physics goes into the EoS?
2) Available data sets:	• Which EoS's are available and how are they used?
3) Comparison and uncertainties:	• How, and how sensitively, do astrophysical simulations depend on the EoS?
4) Strategies for improvements:	• Hypothesis 1: a genereous heuristic extension of the domain of validity fosters scientific progress
5) Userfriendlyness:	• Hypothesis 2: Reliable astrophysical modelling requires long-term availability of reference input physics

Basic phys./astron. question



- Modelling of astro-physical phenomena
- Based on magneto-hydrodynamics and transport processes
- Relying on micro-scopic input physics

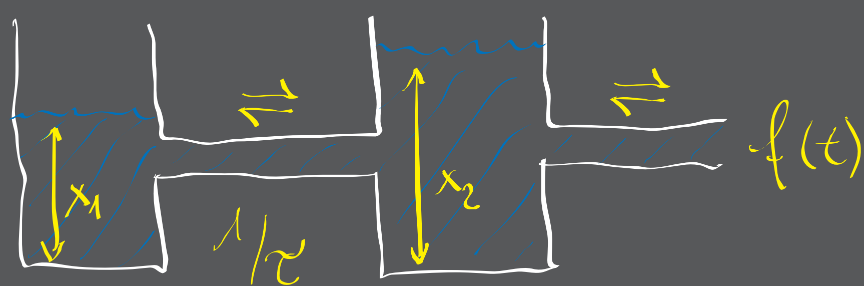
--> Interpreting astro-nomical observations

--> Understanding of astrophysical processes

--> Testing input physics under extreme conditions

Why an equation of state?

Example of system with coupled processes:



$$\frac{\partial x_1}{\partial t} = \frac{x_2 - x_1}{\tau} \qquad \frac{\partial x_2}{\partial t} = \frac{x_1 - x_2}{\tau} + f(t)$$

write equations for sum and difference of x's

$$\frac{\partial}{\partial t}(x_2 + x_1) = f(t)$$

$$\frac{\partial}{\partial t}(x_2 - x_1) = -2\,\frac{x_2 - x_1}{\tau} - f(t)$$

$$\frac{\partial x_2}{\partial t} = \frac{1}{2} f(t)$$

$$x_1 = x_2$$

very fast process: $\tau \to 0$

- processes with time scales much faster than the evolution time scale are set to equilibrium
- the slow processes are evolved numerically
- the equilibrium condition is time-independent and calculated separately
- the equation of state is the most prominent example of this approach!

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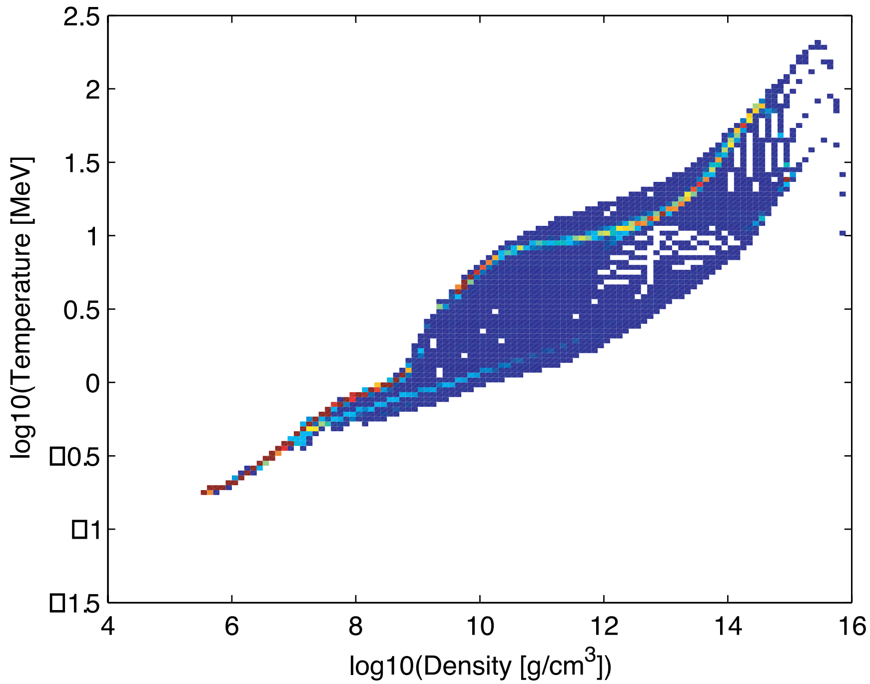
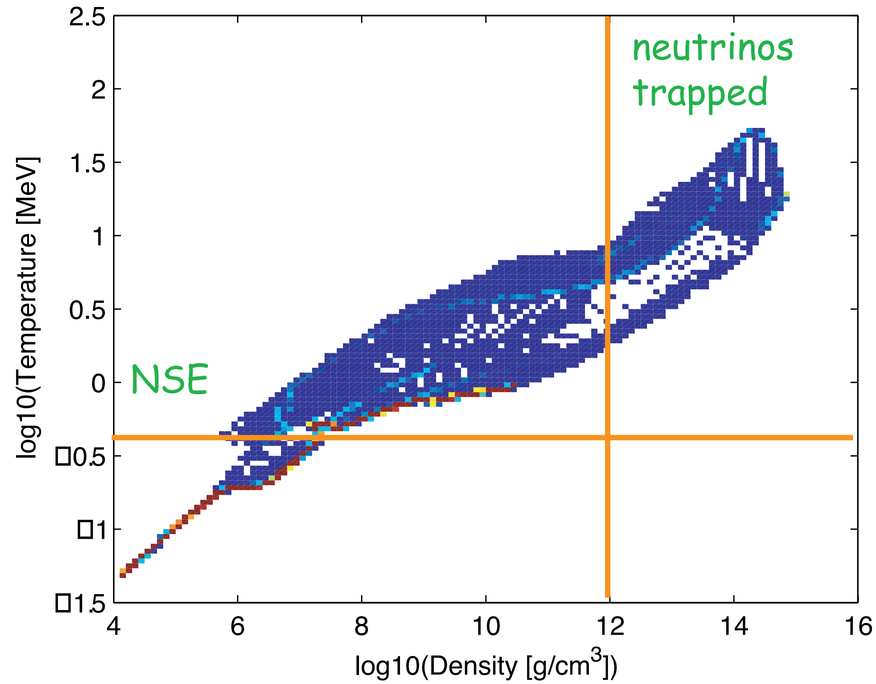
Dream of a global EoS

- thermal equilibrium
- charge neutrality
- nuclear statistical equilibrium
- weak equilibrium
- stationary state
- hydrodstatics

Example for collapse to black hole



Example for explosion



- Different applications require different regimes
- Equilibria do not hold everywhere within one application
- Global EoS probabely stays a dream...

Examples of available EoS's

Name	**Reference**	**Web**	**Feature**	**Form**	**Application**
Ideal gas or polytropic	many	even more	simple & efficient, idealized	Analytical	Collapse test problems
Helmholtz EoS	Timmes & Swesty 1999	www.cococubed .com	ion gas & electrons	Function & Table	FLASH code
Nadyoshin	N. 1974, naucnye informatsii, 32, 33	www.cococubed .com	ion gas & electrons	Analytical	Low density, isolated ions
Wolff-Hillebrandt	1985				Supernovae (out of date)
BCK	BaronCooperstein Kahana, 1985		Liquid drop, very soft EoS	Fitting formula(?)	Supernovae (out of date)
Lattimer & Swesty	L. & S., Nucl. Phys. A, 1991	www.astro.sunysb .edu/dswesty	Liquid drop for one nucleus, NSE	Old function & new table	Supernovae NS Mergers
Shen et al.	S. et al., Prog. Theor. Phys. 1998	www.rcnp.osaka-u.ac.jp/~sumi	Relativistic mean field	Table	Supernovae NS Mergers
Cold bulk nuclear matter	many	many	Hyperons, T=0, beta-equilibrium	simple table	NS matter

Input/Output/Format

Input quantities:
- mass density and temperature
- selection of species
- selection of equilibria
- independent abundances

Output quantities:
- pressure, specific energy, entropy
- information to reconstruct equilibrium (chem. pot.)

Available packages for LS or Shen EoS:

Providers:
http://www.ess.sunysb.edu/lattimer/EOS/main.html
http://www.astro.sunysb.edu/dswesty/
http://www.rcnp.osaka-u.ac.jp/~sumi

Tabulations for neutron stars:
Stephan Rosswog
Max Ruffert

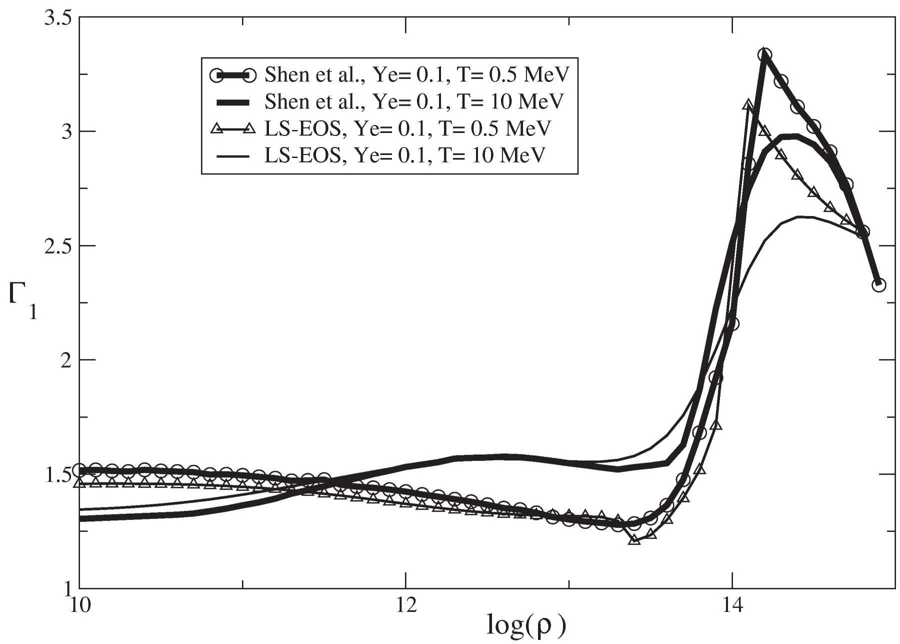
Tabulation for Supernovae:
http://zenith.as.arizona.edu/~burrows/eos.wind.thermal/lseos2.html
http://www.cita.utoronto.ca/~liebend/nuclear/nuclear.html

Format:

- there is no fundamental difference between table and fitting formula
- please: provide table and interpolation algorithm/tool!
- remove arbitrary 'high frequency' noise from data tables?

Dynamical dependence

Example: neutron star mergers

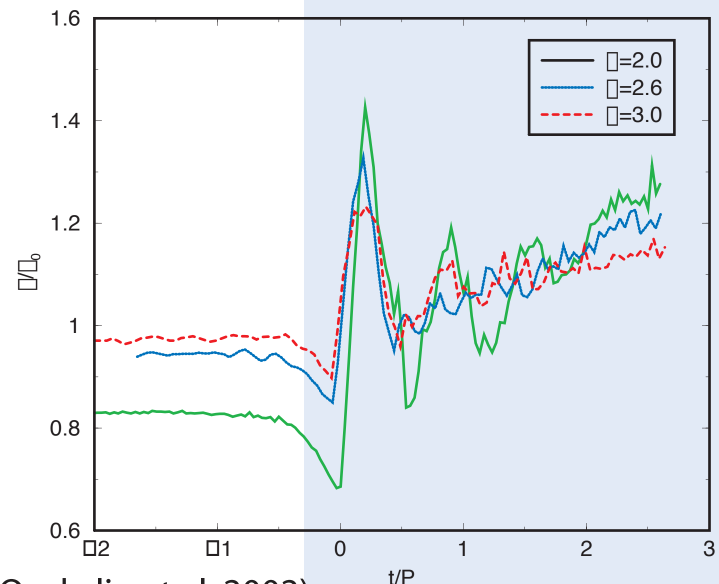


(Rosswog & Davies 2003)

Oscillation frequencies and amplitudes at nuclear density depend on equation of state compressibility

The equation of state determines the dynamics of simulations by

- dynamical quantities (pressure, spec. energy, compressibility)

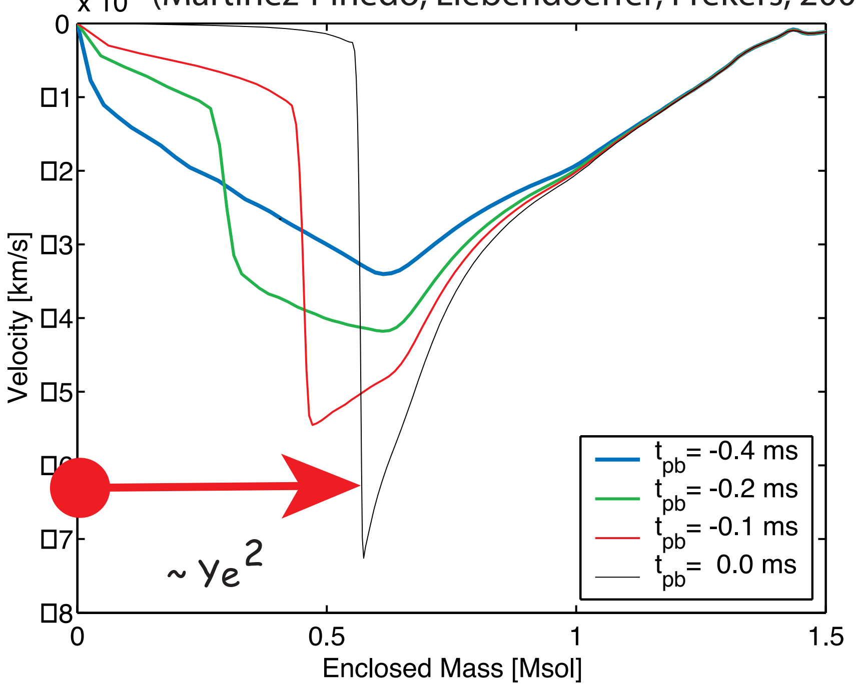


(Oechslin et al. 2002)

Compositional dependence

Example: supernova core collapse

(Martinez-Pinedo, Liebendoerfer, Frekers, 2006)



$$M_{\rm ic} \simeq (\kappa/\kappa_0)^{3/2} M_0,$$

$$\kappa = \frac{\hbar c}{4} (3\pi^2)^{\frac{1}{3}} \left(\frac{Y_e}{m_B}\right)^{\frac{4}{3}}$$

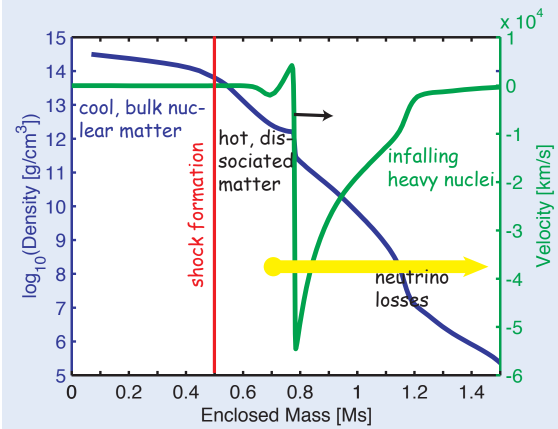
(Goldreich & Weber 1980)

Electron fraction (Ye) set by weak interactions that sensitively depend on composition!

or: neutrino heating is stronger on free nucleons than on nuclei

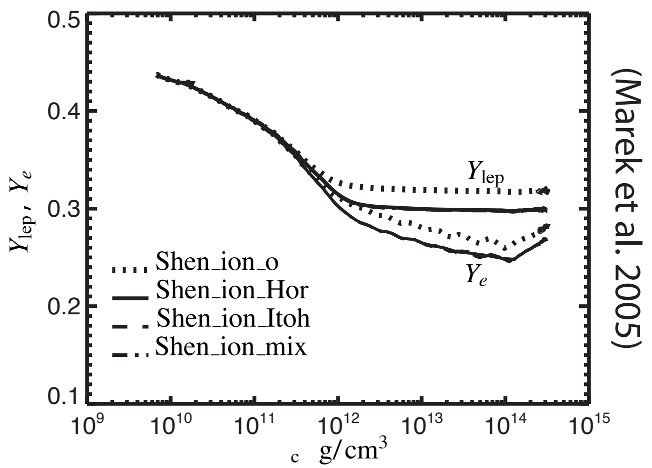
The equation of state determines the dynamics of simulations by

- dynamical quantities, e.g. pressure or compressibility
- composition, which influences e.g. energy release and transfer



Structural dependence

Microscopic: Ion-ion correlations in collapse phase:

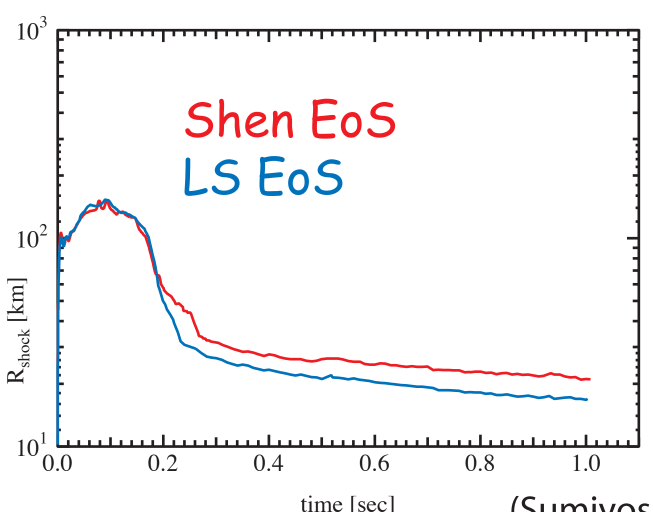


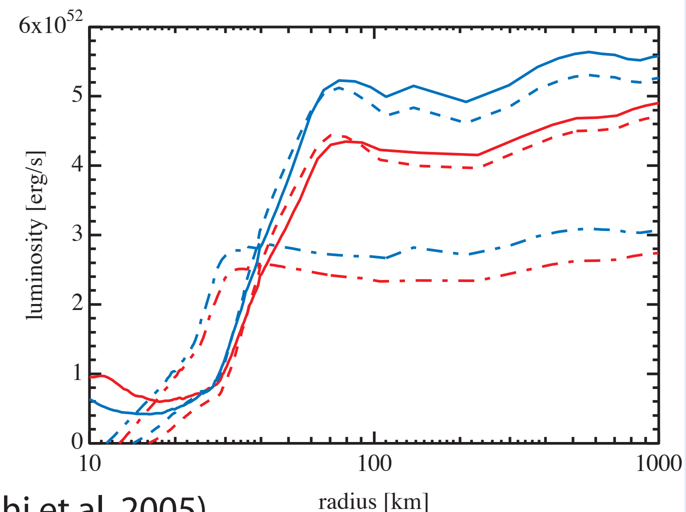
(Marek et al. 2005)

or perhaps more relevant in phase transition from isolated nuclei to bulk nuclear matter?
(e.g. Horowitz et al. 2004, Watanabe et al. 2004)

How far is it from the neutrino spheres?

Macroscopic: Different probing of gravitational potential





(Sumiyoshi et al. 2005)

The equation of state determines the dynamics of simulations by

- dynamical quantities, e.g. pressure or compressibility
- composition, which influences e.g. energy release and transfer
- structure, e.g. microscopically by ion lattice or macroscopically by self-gravitating hydrostatic structure

Secure core with 'halo' data

The cycle of ~~never~~ happening discoveries:

a) A discovery is most likely if a simulation pushes into regimes that have not been simulated before
b) New regime ==> input physics is not in place

~~c) Input physics not in place ==> code crash~~
~~d) Code crash ==> no clue to missing input physics~~
~~e) no clue to missing input physics ==> discovery not made~~

c) Input physics not in place ==> use halo data
d) Using halo data ==> halo flag seen
e) halo flag seen ==> if relevant, improve input physics
f) repeat simulation with improved input physics
g) no halo flag ==> discovery can be made!

Example: $Y_e>0$ in ejecta can only be found in a simulation relying on an EoS that is tabulated beyond $Y_e=0.5$

Suggestion:

- input physics as secure core data, meeting scientific publication quality
- input physics extended by halo data that guarantees at least functionality adjacent to core domain
- an output flag to monitor the usage of halo data

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A product of Monday

Questionnaire

to identify useful input physics for supernova models

Question:	Reference model	Comparative model
1) Does it provide a proven physical improvement?	yes	one yes
2) Does it make a difference?	yes	
3) What is the half-life time T of validity?	> 5 years	> 1 year
4) Is it publicly available and documented?	yes	not yet
5) How much does the computation time increase?	factor 0.5-2	factor 1-10

STANDARD input physics

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For the development of complex numerical models, it is EXTREMELY important to have input physics, that is:

- reasonably accurate (but not too much more...)
- simple and efficient to use
- designated as reference
- long term publicly available (~30 years)

Comparison of spherically symmetric simulations:
Oak Ridge/Basel group and Garching group

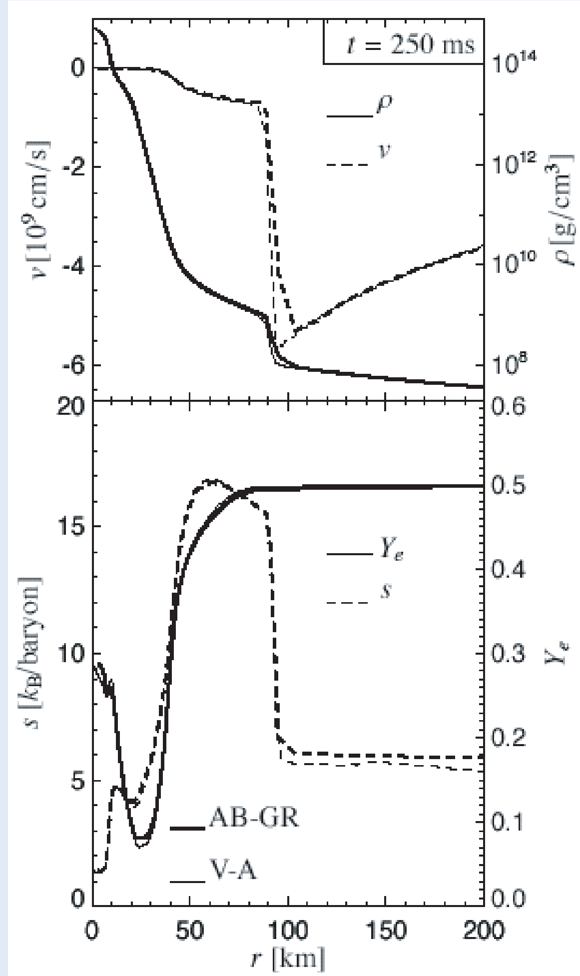
Liebendörfer, Rampp, Janka, Mezzacappa, ApJ 620 (2005)

based on LS-EoS (1991) and Bruenn interactions (1985)

Current supernova simulations are 2D,
future supernova simulations will be 3D

==> no chance to compare and asses simulation quality
without the complete elimination of input physics differences!

excellent agreement:



(Marek et al., A&A 2006)

Conclusions

Physics:
• thermodynamical output mostly ok
• composition important, consistent interface to rates
• microscopic structure interesting at wavelength of transported particles

Strategy:
• secure 'core' data + extensive 'halo' data
• simplest long term reference data + current best choice

Technical:
• data should be smooth to order of physical accuracy
• internal evaluation of equilibrium conditions and external switch to override them
• provide table and interpolating function or parameters and fitting formula
• flag whether output is secure, halo, or nonsense

• equilibrium abundances
• phase transition
• magnetic field

• enables new discoveries
• required to guarantee simulation quality

• keep arbitrary information to minimum

• no fundamental diff. between table & fit!

• keep codes talking...